MAT2705-01/04 07S Quiz 1   Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. Write a differential equation that models the situation: "In a city with a fixed population of P persons, the time rate of change of the number N of those persons infected with a certain contagious disease is proportional to the product of the number who have the disease and the number who do not."

2. $\frac{dy}{dx} = x\,e^{-y}, y(0) = 0.$

a) First find the general solution.

b) Then find the solution which satisfies the given initial condition.

c) Check that your result for part b) actually is a solution by backsubstituting.

▶ solution

1. $\underbrace{\frac{dN}{dt}}_{\text{rate of change of disease count}} \propto \overbrace{\underbrace{N}_{\text{have disease}}(\underbrace{P-N}_{\text{do not}})}^{\text{product}} \rightarrow \boxed{\frac{dN}{dt} = kN(P-N)}$

pendying!

2. a) $\left[\frac{dy}{dx} = xe^{-y}\right] dx\, e^{y}$

$\int e^{y}dy = \int x\,dx$

$\ln\left[e^{y} = \frac{x^2}{2} + C\right]$ ≥0 since LHS ≥0 so no need for abs value here

$\boxed{y = \ln\left(\frac{x^2}{2} + C\right)}$

b) $0 = y(0) = \ln(0 + C) = \ln C$

$e^{0} = e^{\ln C}$

$1 = C$

$\boxed{y = \ln\left(\frac{x^2}{2} + 1\right)}$

backsubstitute into deq

$\frac{dy}{dx} = xe^{-y}$

$\downarrow \qquad \downarrow$

c) $\frac{dy}{dx} = \frac{1}{\frac{x^2}{2}+1}\left(\frac{1}{2}(2x) + 0\right) = \frac{x}{\frac{x^2}{2}+1}$

$\frac{x}{\frac{x^2}{2}+1} = x\,\underbrace{e^{-\ln\left(\frac{x^2}{2}+1\right)}}_{\left(e^{\ln\left(\frac{x^2}{2}+1\right)}\right)^{-1} = \left(\frac{x^2}{2}+1\right)^{-1}}$ no y left.

$= \frac{x}{\left(\frac{x^2}{2}+1\right)}$ ✓