MAT2705-01/04 07S Final Exam Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). **You may use technology for row reductions, determinants, inverses and root finding.** You are encouraged to use technology to check all of your hand results. [As a tip off for silly mistakes, all coefficients in the IVP solutions are simple integers.]

1. $x''+6x'+8x=40\cos(2t)$, $x(0)=0$, $x'(0)=10$.
a) Find the general solution by hand, showing all steps.
b) Find the initial value problem solution by hand, showing all steps.
c) In the actual solution to part b), what are the characteristic times for the exponential terms in your solution and roughly how long does it take for the transient (decaying terms in the solution) to decay to 1 % of its initial value? How does this compare to the period of the steady state solution which is left after the transient decays away [i.e., what is their ratio?]?

2. $x_1'=-9x_1-4x_2$, $x_2'=6x_1+x_2$, $x_3'=-6x_1-4x_2-3x_3$,
$x_1(0)=1$, $x_2(0)=1$, $x_3(0)=0$.
a) Rewrite this system of DEs and its initial conditions in matrix form for the vector variable $\vec{x}=\langle x_1, x_2, x_3\rangle$.
b) Find the general solution, by hand, showing all steps.
c) Find the solution which satisfies the initial conditions, by hand, showing all steps.

3. A two mass, two spring system with parameters $m_1=3$, $m_2=4$, $k_1=9$, $k_2=12$ has the following equations of motion
$x_1''=-7x_1+4x_2$, $x_2''=3x_1-3x_2$, $x_1(0)=8$, $x_2(0)=8$, $x_1'(0)=2$, $x_2'(0)=3$.
a) Rewrite this system of DEs and its initial conditions in matrix form for the vector variable $\vec{x}=\langle x_1, x_2\rangle$.
b) Find the general solution by hand, showing all steps.
c) Find the solution which satisfies the initial conditions, by hand, showing all steps.
d) This has two modes: a "tandem" mode (same sign values of the two unknowns) and an "accordian" mode (opposite sign values of the two unknowns). What are the frequencies and periods of these two oscillating modes respectively (indicate which is which)?
e) Express the solution as an explicit linear combination of the eigenvectors with coefficients $y_1$ and $y_2$. Express each of these coefficient functions exactly in the phase shifted cosine form $A_i\cos(\omega_i t-\delta_i)$.
f) **Optional:** make a diagram of the $x_1$-$x_2$ plane, indicating as arrow vectors the two eigenvectors you found together with the corresponding new coordinate axes $y_1$ and $y_2$. Then add in $\vec{x}(0)$ as an arrow vector, and from its tip draw in $\vec{x}'(0)$ as an arrow vector to represent the initial conditions in your diagram. [If you draw in the parallelogram formed with sides $y_1=\pm A_1$ and $y_2=\pm A_2$, your solution will be confined to this box.]

# ▶ solution

# ▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

Problem 3 helpful grid.