MAT 2705-01/04 07S TEST 2 ANSWERS

① a) $\begin{bmatrix}8\\0\\2\end{bmatrix} = x_1\begin{bmatrix}1\\-1\\1\end{bmatrix} + x_2\begin{bmatrix}2\\0\\1\end{bmatrix} + x_3\begin{bmatrix}3\\1\\0\end{bmatrix} + x_4\begin{bmatrix}4\\2\\1\end{bmatrix}$

$$\begin{bmatrix}1&2&3&4\\-1&0&1&2\\1&1&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}8\\0\\2\end{bmatrix} \rightarrow \begin{bmatrix}1&2&3&4&8\\-1&0&1&2&0\\1&1&0&1&2\end{bmatrix}$$

rref (maple) L L L F

$$\begin{bmatrix}1&0&0&-2&2\\0&1&0&3&0\\0&0&1&0&2\end{bmatrix} \rightarrow \begin{array}{l} x_1 - 2x_4 = 2 \rightarrow x_1 = 2+2t \\ x_2 + 3x_4 = 0 \rightarrow x_2 = -3t \\ x_3 = 2 \rightarrow x_3 = 2 \end{array}$$

$x_4 = t \rightarrow x_4 = t$

$\langle x_1, x_2, x_3, x_4\rangle = \langle 2+2t, -3t, 2, t\rangle$

$$\boxed{\begin{bmatrix}8\\0\\2\end{bmatrix} = (2+2t)\begin{bmatrix}1\\-1\\1\end{bmatrix} + -3t\begin{bmatrix}2\\0\\1\end{bmatrix} + 2\begin{bmatrix}3\\1\\0\end{bmatrix} + t\begin{bmatrix}4\\2\\1\end{bmatrix}}$$

$$= \begin{bmatrix}(2+2t) - 6t + 6 + 4t\\ -(2+2t) + 0 + 2 + 2t\\ (2+2t) - 3t + 0 + t\end{bmatrix} = \begin{bmatrix}8\\0\\2\end{bmatrix} \checkmark$$

b) replace $\langle 8,0,2\rangle$ by $\langle 0,0,0\rangle$ get

$\langle x_1, x_2, x_3, x_4\rangle = \langle 2t, -3t, 0, t\rangle = t\langle 2, -3, 0, 1\rangle$ ← coeffs of single ind. relationship

$$\boxed{2\begin{bmatrix}1\\-1\\1\end{bmatrix} - 3\begin{bmatrix}2\\0\\1\end{bmatrix} + 0\begin{bmatrix}3\\1\\0\end{bmatrix} + 1\begin{bmatrix}4\\2\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}}$$

② a) $800y'' + 60y' + y = 0$

$y = e^{rx} \rightarrow (800r^2 + 60r + 1)e^{rx} = 0$

$(800r^2 + 60r + 1) = 0$ solve, maple

$r = -\frac{1}{20}, -\frac{1}{40}$ $\quad (-.05, -.025)$

$e^{rx} = e^{-\frac{x}{20}}, e^{-\frac{x}{40}}$

gen soln. $\boxed{y = c_1 e^{-\frac{x}{20}} + c_2 e^{-\frac{x}{40}}}$

b) $y' = -\frac{c_1}{20}e^{-\frac{x}{20}} - \frac{c_2}{40}e^{-\frac{x}{40}}$

$y(0) = c_1 + c_2 = 4$

$y'(0) = -\frac{c_1}{20} - \frac{c_2}{40} = 1$

$$\begin{bmatrix}1&1\\-\frac{1}{20}&-\frac{1}{40}\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$$

$$\begin{bmatrix}c_1\\c_2\end{bmatrix} = \frac{1}{-\frac{1}{40}+\frac{1}{20}}\begin{bmatrix}-\frac{1}{40}&-1\\\frac{1}{20}&1\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix} = +40\begin{bmatrix}-\frac{1}{10}-1\\\frac{1}{5}+1\end{bmatrix}$$

$$= 40\begin{bmatrix}-11/10\\6/5\end{bmatrix} = \begin{bmatrix}-44\\48\end{bmatrix}$$

$$\boxed{y = -44e^{-\frac{x}{20}} + 48e^{-\frac{x}{40}}}$$

c) $y' = +\frac{44}{20}e^{-\frac{x}{20}} - \frac{48}{40}e^{-\frac{x}{40}} = 0$

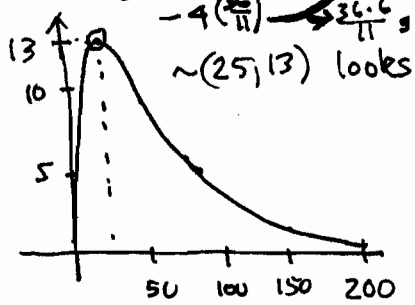
$\left[44e^{-\frac{x}{20}} = 24e^{-\frac{x}{40}}\right]e^{\frac{x}{20}}$

$\frac{11}{6} = \frac{44}{24} = e^{\frac{x}{40}}$, $x = \boxed{40\ln\frac{11}{6} \approx 24.25}$

$y = -44e^{-2\ln\frac{11}{6}} + 48e^{-\ln\frac{11}{6}}$

$= -44\left(\frac{6}{11}\right)^2 + 48\left(\frac{6}{11}\right) \overset{\text{lazy! maple grouping!!}}{=} \frac{144}{11} \approx \boxed{13.090}$

$-4\left(\frac{36}{11}\right) \rightarrow \frac{36\cdot 6}{11}$



~(25, 13) looks good!

③ a) $W = \begin{bmatrix}\sin x & \sin^3 x & \sin 3x \rightarrow\\ \cos x & 3\sin^2 x\cos x & 3\cos 3x\\ -\sin x & 6\sin x\cos^2 x - 3\sin^3 x & -9\sin 3x\end{bmatrix}$

$$A = W|_{x=\pi/2} = \begin{bmatrix}1&1&-1\\0&0&0\\-1&-3&9\end{bmatrix}$$

$\sin \pi/2 = 1$, $\cos \pi/2 = 0$, $\sin \frac{3\pi}{2} = -1$, $\cos \frac{3\pi}{2} = 0$

b) $A\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$$\begin{bmatrix}1&1&-1&0\\0&0&0&0\\-1&-3&9&0\end{bmatrix} \rightarrow \text{maple} \rightarrow \begin{bmatrix}1&0&3&0\\0&1&-4&0\\0&0&0&0\end{bmatrix}$$

L L F

$x_1 + 3x_3 = 0$, $x_2 - 4x_3 = 0$

$x_1\ x_2\ x_3$

t

$x_1 = -3t$, $x_2 = 4t$, $x_3 = t$

$\boxed{\langle x_1, x_2, x_3\rangle = t\langle -3, 4, 1\rangle}$

therefore $-3\sin x + 4\sin^3 x + \sin 3x = 0$

or $\boxed{\sin 3x = 3\sin x - 4\sin^3 x}$

c) $\sin\frac{3\pi}{6} = 3\sin\frac{\pi}{6} - 4\sin^3\frac{\pi}{6} = \frac{3}{2} - \frac{4}{8}$

$\sin\frac{\pi}{2} = 1$, $\frac{1}{2}$, $\left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - \frac{1}{2}$

$= 1$ ✓