MAT2705-01/04 07S Test 2 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions and root finding.**

1. a) Express the vector $\langle 8, 0, 2\rangle$ in terms of the four vectors $\{\langle 1,-1, 1\rangle, \langle 2, 0, 1\rangle, \langle 3, 1, 0\rangle, \langle 4, 2, 1\rangle\}$ and then check that the linear combination that you find evaluates to the original vector. Be sure to state clearly all steps in the process.
b) From your work for part a), state the linear relationship which exists among these four vectors. Explain.

2. $800\,y''+60\,y'+y=0,\ \ y(0)=4, y'(0)=1$.
a) Find the general solution $y(x)$ of this differential equation by hand (no decimals).
b) Find the particular solution $y(x)$ which satisfies the initial conditions, by hand, using matrix techniques once you state the equivalent linear system of equations.
c) Your solution should have a single local (and global) maximum for some $x > 0$. Make an ***appropriately*** chosen window to plot your solution and estimate the location of this peak. Then find its (x, y) values exactly using calculus and give accurate approximate numerical values; if you cannot find it exactly, at least find accurate numerical values. Do your numbers look right compared to your plot? Make a completely labeled sketch that conveys this information.

3. It is known that the three functions $\{\sin(x), \sin(x)^3, \sin(3\,x)\}$ satisfy a triple angle identity which enables $\sin(3\,x)$ to be expressed in terms of the first two odd powers of $\sin(x)$.
a) Evaluate the Wronskian matrix of these three functions (which form the first row, their derivatives the second row, and their second derivatives the third row), and then evaluate this result at $x=\pi/2$ to obtain a matrix A.
b) Find the general solution of the linear system $A\,\mathbf{x}=\mathbf{0}$ for the coefficients $\mathbf{x}=\langle x1, x2, x3\rangle$ of a linear relationship among these three functions.
c) Use it to express $\sin(3\,x)$ in terms of the other two functions.
d) Check that it actually holds for $x=\pi/6$ where you know the exact values of these functions.

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: