MAT2705-01/06 07F Quiz 2 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $x y' + y = 1,\ \ y(1) = -2$.

a) First find the general solution of this DE using the separable technique.

b) Then find the general solution of this DE using the linear technique.

c) Do they agree? Explain. Which technique was clearly easier?

d) Now find the solution which satisfies the initial condition $y(1) = -2$.

e) **Optional** No Credit: Check that your result for part b) is actually a solution of the DE by backsubstituting into the DE.

▶ solution

① a) $xy' + y = 1 \xrightarrow{\text{standard form}} y' = \frac{1-y}{x} \xrightarrow{\text{separable}} \frac{dy}{dx} = \frac{1-y}{x} \xrightarrow{\text{separate and int.}} \int \frac{dy}{1-y} = \int \frac{dx}{x}$

$e^{\left[\underbrace{-\ln|1-y|}_{\text{u-sub sign}} = \ln|x| + C_1\right]} \rightarrow \frac{1}{|1-y|} = e^{\ln|x|} e^{C_1} = |x| e^{C_1}$

$\frac{1}{1-y} = \underbrace{(\pm e^{C_1})}_{= C_2} x \rightarrow 1 - y = \frac{1}{C_2 x}$

$y = 1 \underbrace{- \frac{1}{C_2}}_{= C} \frac{1}{x} = 1 + \frac{C}{x}$

gen soln : $\boxed{y = 1 + \frac{C}{x}}$

b) $\frac{dy}{dx} = \frac{1}{x} - \frac{y}{x} \xrightarrow{\text{standard form}} \left[\frac{dy}{dx} + \frac{1}{x} y = \frac{1}{x}\right] \rightarrow x\left[\frac{dy}{dx} + \frac{1}{x} y\right] = x \cdot \frac{1}{x} \rightarrow \frac{d}{dx}(xy) = 1$

$e^{\int \frac{1}{x} dx = \ln|x|}$ $e = |x| = \pm x$

I.F.: x

$xy = x + C_3$

$y = \frac{x + C_3}{x} = 1 + \frac{C_3}{x}$, $C_3 = C$

gen soln : $\boxed{y = 1 + \frac{C}{x}}$ agrees with a).

c) Obviously they agree. The linear technique involves less algebra.

d) $-2 = 1 + \frac{C}{1} \rightarrow C = -3 \rightarrow \boxed{y = 1 - \frac{3}{x}}$

e) $\frac{dy}{dx} = \frac{d}{dx}(1 - \cancel{3}^{C} x^{-1}) = 0 - \cancel{3}^{C}(-1)x^{-2} = \frac{\cancel{3}^{C}}{x^2}$

oops, not paying attention!

$x\left(\frac{\cancel{3}^{C}}{x^2}\right) + \left(1 - \frac{\cancel{3}^{C}}{x}\right) \stackrel{?}{=} 1$

$\frac{\cancel{C3}}{\cancel{x}} + 1 - \frac{\cancel{3}^{C}}{\cancel{x}} = 1$

$1 = 1$ ✓