MAT 2705-01/06 07F Takehome TEST 3 Answers (1)

1) a) $y'' + 4y' + 13y = F$

$y_h = e^{rt} \nearrow \searrow (\underbrace{r^2+4r+13}_{=0})e^{rt} = 0$

$r = \frac{-4 \pm \sqrt{16-4\cdot 13}}{2} = -2 \pm 3i$

$y_h \approx e^{(-2\pm 3i)t} = e^{-2t}(\cos 3t \pm i \sin 3t)$

$y_h = e^{-2t}(c_1 \cos 3t + c_2 \sin 3t)$

$F(t) = 1 = 1e^{0t} \leftrightarrow r=0: \quad D(1) = 0$

$y_p = c_3$

$y_p'' + 4y_p' + 13y_p = 0 + 0 + 13(c_3) = 1$

$c_3 = 1/13$

$y = y_h + y_p = e^{-2t}(c_1 \cos 3t + c_2 \sin 3t) + \frac{1}{13}$

$y' = -2e^{-t}(c_1 \cos 3t + c_2 \sin 3t) + e^{-t}(-3c_1 \sin 3t + 3c_2 \cos 3t)$

$y(0) = c_1 + \frac{1}{13} = 0 \rightarrow c_1 = -1/13$

$y'(0) = -2c_1 + 3c_2 = 0 \rightarrow c_2 = \frac{2}{3}c_1 = -\frac{2}{39}$

$$\boxed{y = \frac{1}{13}\left[e^{-2t}(-\cos 3t - \tfrac{2}{3}\sin 3t) + 1\right]}$$

$\lim_{t\to\infty} y = \frac{1}{13} = y_\infty$

b) $F(t) = 1 - e^{-2t}$

1 → as before; e^{-2t} → $r=-2$, $(D+2)e^{-2t} = 0$

b) $y_p = c_3 + c_4 e^{-2t}$

$y_p' = -2c_4 e^{-2t}$

$y_p'' = 4c_4 e^{-2t}$

$y_p'' + 4y_p' + 13y_p = [\underbrace{4 + 4(-2) + 13}_{9}]c_4 e^{-2t} + 13c_3 = 1 - e^{-2t}$

$13c_3 = 1, \quad 9c_4 = -1 \rightarrow c_3 = \frac{1}{13}, \quad c_4 = \frac{-1}{9}$

$y = e^{-2t}(c_1 \cos 3t + c_2 \sin 3t) + \frac{1}{13} - \frac{1}{9}e^{-2t}$

$y' = -2e^{-2t}(c_1 \cos 3t + c_2 \sin 3t) + 0 + \frac{2}{9}e^{-2t}$
$\quad + e^{-2t}(-3c_1 \sin 3t + 3c_2 \cos 3t)$

$y(0) = c_1 + \frac{1}{13} - \frac{1}{9} = 0 \rightarrow c_1 = \frac{1}{9} - \frac{1}{13} = \frac{13-9}{117} = \boxed{\frac{4}{117}}$

$y'(0) = -2c_1 + 3c_2 + \frac{2}{9} = 0 \rightarrow c_2 = \frac{1}{3}(2c_1 - \frac{2}{9})$

$\quad = \frac{1}{3}\left(\frac{8}{9\cdot 13} - \frac{2}{9}\right) = \frac{2}{27}\left(\frac{4}{13} - 1\right) = \frac{-2}{3(13)} = \boxed{\frac{-2}{39}}$

$$\boxed{y = \frac{e^{-2t}}{13}\left(\frac{4}{9}\cos 3t - \frac{2}{3}\sin 3t\right) + \frac{1}{13} - \frac{1}{9}e^{-2t}}$$

$y_\infty = \lim_{t\to\infty} y = \frac{1}{13}$

c) see plots at end.

d) $F(t) = \sin 3t$

$13\ (y_p = c_3 \cos 3t + c_4 \sin 3t)$

$4\ (y_p' = -3c_3 \sin 3t + 3c_4 \cos 3t)$

$1\ (y_p'' = -9c_3 \cos 3t - 9c_4 \sin 3t)$

$y_p'' + 4y_p' + 13y_p = [(13-9)c_3 + 12c_4]\cos 3t + [-12c_3 + (13-9)c_4]\sin 3t$
$= (4c_3 + 12c_4)\cos 3t + (-12c_3 + 4c_4)\sin 3t = \sin 3t$

$4c_3 + 12c_4 = 0$
$-12c_3 + 4c_4 = 1$

$\begin{bmatrix} 4 & 12 \\ -12 & 4 \end{bmatrix}\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{\underbrace{16+144}_{=160}}\begin{bmatrix} 4 & -12 \\ 12 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \frac{1}{160}\begin{bmatrix} -12 \\ 4 \end{bmatrix} = \begin{bmatrix} -3/40 \\ 1/40 \end{bmatrix}$

$y = e^{-2t}(c_1 \cos 3t + c_2 \sin 3t)$
$\quad + \frac{1}{40}[-3\cos 3t + \sin 3t]$

$y' = -2e^{-2t}(c_1 \cos 3t + c_2 \sin 3t) + e^{-t}(-3c_1 \sin 3t + 3c_2 \cos 3t)$
$\quad + \frac{1}{40}(9 \sin 3t + 3\cos 3t)$

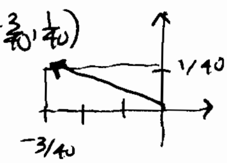
$y(0) = c_1 - 3/40 = 0 \rightarrow c_1 = 3/40$

$y'(0) = -2c_1 + 3c_2 + 3/40 = 0$

$\hookrightarrow c_2 = \frac{1}{3}(2c_1 - 3/40) = \frac{1}{3}\left(\frac{3}{40}\right) = \frac{1}{40}$

$$\boxed{y = \frac{1}{40}\left[e^{-2t}(3\cos 3t + \sin 3t) + (-3\cos 3t + \sin 3t)\right]}$$

$(-\frac{3}{40}, \frac{1}{40})$



$A = \frac{\sqrt{9+1}}{40} = \frac{\sqrt{10}}{40} \approx 0.079$

$\delta = \pi - \arctan(\frac{1}{3})$

$\approx 2.82 \approx 161.6° \approx 0.45$ cycles

e) $y'' + 4y' + 13y$
$\uparrow 1 \quad \uparrow k_0 \quad \uparrow \omega_0^2$

$\tau_0 = 1/k_0 = 1/4$

$\omega_0 = \sqrt{13} \approx 3.61$

$Q = \frac{\sqrt{13}}{4} \approx 0.901$

f) $13\ [y_p = c_3 \cos\omega t + c_4 \sin\omega t]$

$4\ [y_p' = -\omega c_3 \sin\omega t + \omega c_4 \cos\omega t]$

$1\ [y_p'' = -\omega^2 c_3 \cos\omega t - \omega^2 c_4 \sin\omega t]$

$y_p'' + 4y_p' + 13y_p = [(13-\omega^2)c_3 + 4\omega c_4]\cos\omega t$
$\quad + [-4\omega c_3 + (13-\omega^2)c_4]\sin\omega t = \sin\omega t$

$\begin{bmatrix} 13-\omega^2 & 4\omega \\ -4\omega & 13-\omega^2 \end{bmatrix}\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{\underbrace{(13-\omega^2)^2 + 16\omega^2}_{\mathcal{D}}}\begin{bmatrix} 13-\omega^2 & -4\omega \\ 4\omega & 13-\omega^2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \frac{1}{\mathcal{D}}\begin{bmatrix} -4\omega \\ 13-\omega^2 \end{bmatrix}$

g) $A(\omega) = \sqrt{c_3^2 + c_4^2} = \frac{\sqrt{(13-\omega^2)^2 + 16\omega^2}}{\mathcal{D}} = \frac{1}{\sqrt{(13-\omega^2)^2 + 16\omega^2}}$

$$\boxed{y_{ss} = y_p = \frac{1}{\sqrt{(13-\omega^2)^2 + 16\omega^2}}\left[-4\omega \cos\omega t + (13-\omega^2)\sin\omega t\right]}$$

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1 a) $A'(\omega) = -\frac{1}{2}\left((13-\omega^2)^2+16\omega^2\right)^{-3/2}\underbrace{\left[2(13-\omega^2)(-2\omega)+16(2\omega)\right]}_{=0} = 0$

$4\omega\left[(\omega^2-13)+8\right] = 0 \to \omega = 0, \sqrt{5}\ (\ge 0).$

$\boxed{\omega_p = \sqrt{5} \approx 2.236}$

$A(\omega_p) = \frac{1}{\sqrt{(13-5)^2+16\cdot 5}} = \frac{1}{\sqrt{64+80}} = \frac{1}{\sqrt{144}} = \boxed{\frac{1}{12} \approx 0.083}$

$A(0) = \frac{1}{\sqrt{(13)^2}} = \frac{1}{13}$ $\qquad \boxed{\frac{A(\omega_p)}{A(0)} = \frac{1/12}{1/13} = \frac{13}{12} \approx 1.083}$

$A(\omega_0) = \frac{1}{\sqrt{0+16\cdot 13}} = \boxed{\frac{1}{\sqrt{208}} \approx 0.0693}$ for plot see below

$A(3) = \left[(13-9)^2+16\cdot 9\right]^{-1/2} = \frac{1}{4\sqrt{10}} = \frac{\sqrt{10}}{40}$ ✓ agrees with part d)

2 a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \underbrace{\begin{bmatrix} -3 & 0 & 0 \\ 3 & -2 & 0 \\ 0 & 2 & -1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$

$\to \underline{x}' = A\underline{x}$

b) $|A-\lambda I| = \begin{vmatrix} -3-\lambda & 0 & 0 \\ 3 & -2-\lambda & 0 \\ 0 & 2 & -1-\lambda \end{vmatrix} = -(\lambda+1)(\lambda+2)(\lambda+3) = 0$

$\lambda = -1, -2, -3$

$\lambda=-1$: $A+I = \begin{bmatrix} -2 & 0 & 0 \\ 3 & -1 & 0 \\ 0 & 2 & 0 \end{bmatrix} \to \begin{array}{ccc} L & L & F \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_3 = t \quad x_1 = 0, x_2 = 0 \to \underline{x} = \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} = t\underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\vec{b}_1}$

$\lambda=-2$; $A+2I = \begin{bmatrix} -1 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix} \to \begin{array}{ccc} L & L & F \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_3 = t, \ x_1 = 0, \ x_2 = -\frac{1}{2}t \qquad \underline{x} = \begin{bmatrix} 0 \\ -t/2 \\ t \end{bmatrix} = t\underbrace{\begin{bmatrix} 0 \\ -1/2 \\ 1 \end{bmatrix}}_{\vec{b}_2}$

$\lambda=-3$: $A+3I = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix} \to \begin{array}{ccc} L & L & F \end{array} \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_3 = t, \ x_1 = t/3, \ x_2 = -t \qquad \underline{x} = \begin{bmatrix} t/3 \\ -t \\ t \end{bmatrix} = t\underbrace{\begin{bmatrix} 1/3 \\ -1 \\ 1 \end{bmatrix}}_{\vec{b}_3}$

$\lambda = -1, -2, -3$

$B = \begin{bmatrix} 0 & 0 & 1/3 \\ 0 & -1/2 & -1 \\ 1 & 1 & 1 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 3 & 2 & 1 \\ -6 & -2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \quad A_B = B^{-1}AB = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

b) $\underline{x} = B\underline{y}, \ \underline{y} = B^{-1}\underline{x}$

$\underline{y}' = A_B\underline{y}$

$y_1' = -y_1 \qquad y_1 = c_1e^{-t}$

$y_2' = -2y_2 \qquad y_2 = c_2e^{-2t}$

$y_3' = -3y_3 \qquad y_3 = c_3e^{-3t}$ $\qquad$ GEN SOL:

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/3 \\ 0 & -1/2 & -1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} c_1e^{-t} \\ c_2e^{-2t} \\ c_3e^{-3t} \end{bmatrix} = \begin{bmatrix} \frac{1}{3}c_1e^{-t} \\ -\frac{1}{2}c_2e^{-2t} - c_3e^{-3t} \\ c_1e^{-t}+c_2e^{-2t}+c_3e^{-3t} \end{bmatrix}$

c) $\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = B\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ -6 & -2 & 0 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 27 \\ -54 \\ 27 \end{bmatrix}$

slowest to decay $\tau = 1$, $5\tau = 5$ for plot

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9e^{-t} \\ +27e^{-2t} - 27e^{-3t} \\ 27e^{-t} - 54e^{-2t} + 27e^{-3t} \end{bmatrix}$ IVP soln

d) x_1 is max at $\boxed{t=0}$: $x_1(0) = \boxed{9}$

$x_2' = \left[-2(27)e^{-2t} + 3(27)e^{-3t} = 0\right]\frac{e^{3t}}{27}$

$-2e^t + 3 = 0, \ e^t = 3/2$

$\boxed{t = \ln 3/2 \approx 0.405}$

$x_2(\ln 3/2) = 27\left(e^{-2\ln\frac{3}{2}} - e^{-3\ln\frac{3}{2}}\right)$

$= 27\left[\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3\right] = 27\left(\frac{4}{9}\right)\left(1-\frac{2}{3}\right) = \boxed{4}$ ✓

$x_3' = 27\left(-e^{-t} + 4e^{-2t} - 3e^{-3t}\right)$ let Maple solve!

clever observation not necessary

$= -27e^{-3t}\left(e^{2t} - 4e^t + 3\right) = 0$

$e^t = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} = 3, 1$

$\boxed{t = \ln 3, \ln 1 = 0}$ $\qquad \boxed{t_{max} = \ln 3 \approx 1.10}$

$x_3(\ln 3) = 27\left(e^{-\ln 3} - 2e^{-2\ln 3} + e^{-3\ln 3}\right)$

$= 27\left(\frac{1}{3} - \frac{2}{9} + \frac{1}{27}\right) = 27\left(\frac{9-6+1}{27}\right) = \boxed{4}$ ✓

for plot see below

$x_1 = .09 \to t \approx 1.535$	$x_1 = .03 \to t = 1.901$
$x_2 = .09 \to t = 2.821$	$x_2 = .03 \to t = 3.384$
$x_3 = .09 \to t = 5.697$	$x_3 = .03 \to t = 6.800$

③ a) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \underbrace{\begin{bmatrix} 1 & -5 \\ 1 & 3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\underline{x}' = A\underline{x}$

b) $|A-\lambda I| = \begin{vmatrix} 1-\lambda & -5 \\ 1 & 3-\lambda \end{vmatrix} = (\lambda-1)(\lambda-3)+5 = \lambda^2-4\lambda+8=0$

$\lambda = \frac{4\pm\sqrt{16-4\cdot 8}}{2} = 2\pm\sqrt{4-8} = 2\pm 2i$

$\lambda=2+2i$: $A-\lambda I = \begin{bmatrix} 1-(2+2i) & -5 \\ 1 & 3-(2+2i) \end{bmatrix}$

$= \begin{bmatrix} -1-2i & -5 \\ 1 & 1-2i \end{bmatrix} \to \begin{bmatrix} 1 & 1-2i \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_2 = t,\ x_1 = -(1-2i)t \quad \underline{x} = t\begin{bmatrix} -1+2i \\ 1 \end{bmatrix}\ \vec{b}_1$

$b_2 = \overline{b_1} = \begin{bmatrix} -1-2i \\ 1 \end{bmatrix}$

$B = \begin{bmatrix} -1+2i & -1-2i \\ 1 & 1 \end{bmatrix} \quad B^{-1}AB = \begin{bmatrix} 2+2i & 0 \\ 0 & 2-2i \end{bmatrix}$

$\lambda = 2+2i,\ 2-2i$

$\underline{x} = B\underline{y},\ \underline{y} = B^{-1}\underline{x}$

$\underline{y}' = A_B\,\underline{y} \quad y_1' = (2+2i)y_1 \quad y_1 = c_1 e^{(2+2i)t}$
$\qquad\qquad y_2' = (2-2i)y_2 \quad y_2 = c_2 e^{(2-2i)t}$

$x = \begin{bmatrix} -1+2i & -1-2i \\ 1 & 1 \end{bmatrix}\begin{bmatrix} c_1 e^{(2+2i)t} \\ c_2 e^{(2-2i)t} \end{bmatrix}$

$= c_1 e^{(2+2i)t}\begin{bmatrix} -1+2i \\ 1 \end{bmatrix} + c_2 e^{(2-2i)t}\begin{bmatrix} -1-2i \\ 1 \end{bmatrix}$

$= e^{2t}(\cos 2t + i\sin 2t)\begin{bmatrix} -1+2i \\ 1 \end{bmatrix}$

$= e^{2t}\begin{bmatrix} -\cos 2t - 2\sin 2t + i(2\cos 2t - \sin 2t) \\ \cos 2t + i\sin 2t \end{bmatrix}$

$= e^{2t}\begin{bmatrix} -\cos 2t - 2\sin 2t \\ \cos 2t \end{bmatrix} + i\, e^{2t}\begin{bmatrix} 2\cos 2t - \sin 2t \\ \sin 2t \end{bmatrix}$

$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c_1 e^{2t}\begin{bmatrix} -\cos 2t - 2\sin 2t \\ \cos 2t \end{bmatrix} + c_2 e^{2t}\begin{bmatrix} 2\cos 2t - \sin 2t \\ \sin 2t \end{bmatrix}}$

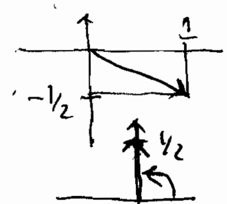
$\begin{pmatrix}\text{gen} \\ \text{soln}\end{pmatrix} = \boxed{\begin{bmatrix} e^{2t}[(-c_1+2c_2)\cos 2t - (2c_1+c_2)\sin 2t] \\ e^{2t}[c_1\cos 2t + c_2 \sin 2t] \end{bmatrix}}$

c) $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -c_1+2c_2 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \to c_1 = 0,\ c_2 = 1/2$

$-c_1+2c_2 = 1$
$-2c_1 - c_2 = -1/2$

$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e^{2t}(\cos 2t - \frac{1}{2}\sin 2t) \\ \frac{1}{2}e^{2t}\sin 2t \end{bmatrix}}$ IVP soln

d)



$A_1 = \sqrt{1^2 + (\frac{1}{2})^2} = \frac{\sqrt{5}}{2} \approx 1.118$

$\delta_1 = -\arctan(1/2) \approx -0.464 \approx -26.6°$

$A_2 = 1/2,\ \delta = \pi/2 = 90°$

$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{5}}{2}e^{2t}(\cos(2t + \arctan(1/2))) \\ \frac{1}{2}e^{2t}\cos(2t - \pi/2) \end{bmatrix}}$

f) $\delta_2 - \delta_1 = \frac{\pi}{2} + \arctan(\frac{1}{2}) \approx 2.034 \approx 116.6°$

≈ 0.324 cycles $(= (\delta_2-\delta_1)/2\pi)$

x_1 is shifted to the left ($\delta_1 < 0$) while x_2 is shifted to the right ($\delta_2 > 0$), so x_1 has its peaks first in time.

④ a) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \underbrace{\begin{bmatrix} -6 & 4 \\ 1 & -6 \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$|A-\lambda I| = \begin{vmatrix} -6-\lambda & 4 \\ 1 & -6-\lambda \end{vmatrix} = (\lambda+6)^2 - 4 = \lambda^2 + 12\lambda + 32 = 0$

$\lambda = \frac{-12\pm\sqrt{12^2-4\cdot 4\cdot 8}}{2} = -6\pm 2$

$= -8, -4 \to -4, -8$ dec order

$\lambda = -8$:
$A + 8I = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} x_1 = -2t \\ x_2 = t \end{matrix} \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t\underbrace{\begin{bmatrix} -2 \\ 1 \end{bmatrix}}_{\vec{b}_2}$

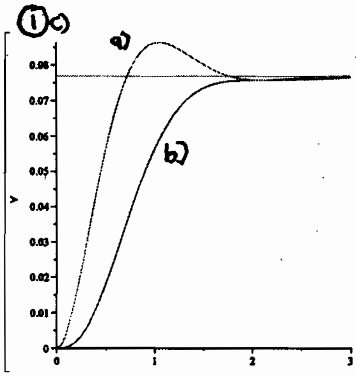
$\lambda = -4$
$A + 4I = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} x_1 = 2t \\ x_2 = t \end{matrix} \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t\underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{\vec{b}_1}$

$\lambda = -4, -8$

$B = \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

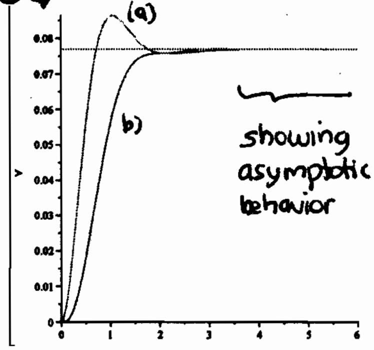
$B^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 3 \end{bmatrix} \to \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix}$

① c)



See maple worksheet on line for full size plots

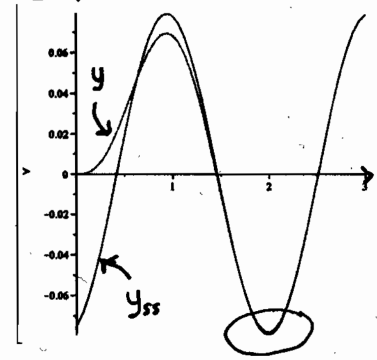
① c)



showing asymptotic behavior

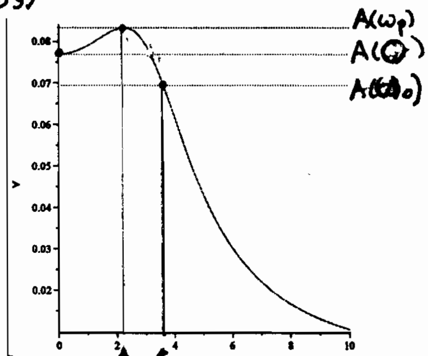
① d)

4



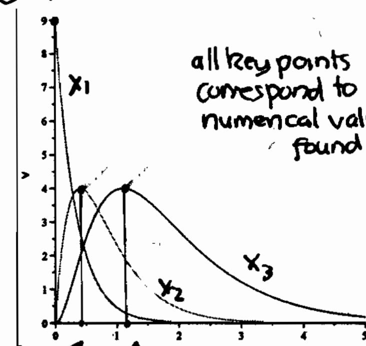
slight difference apparent still

① g)



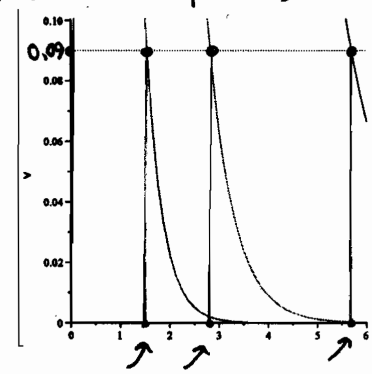
corresponds to numerical values found

② d)



all key points correspond to numerical values found

② d) closeup along axis

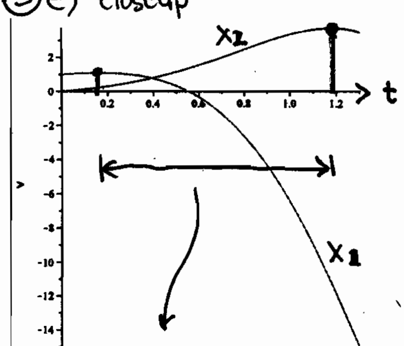


corresponds to numerical values found

$x = \underbrace{A(t)}\cos(\omega t - \delta)$

envelope curves: $x = \pm A(t)$

③ e) closeup

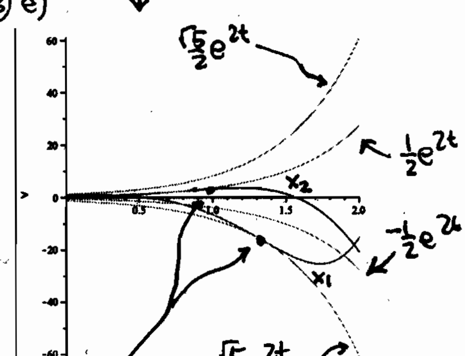


$\Delta t \approx 1.20 - 0.15 \approx 1.05$

$\omega\Delta t = 2\Delta t \approx 2.1$ radians

comparable to calculated value 2.034 !

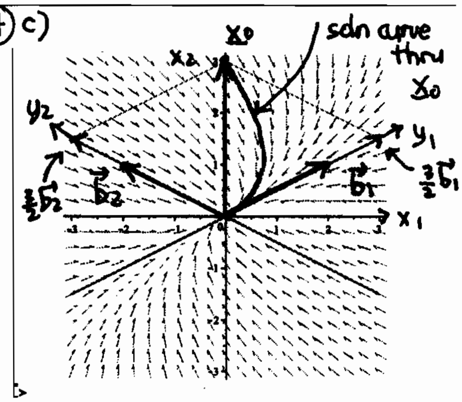
③ e)



x_1, x_2 nicely touch envelopes at dots

$(\tau = \frac{1}{2},\ 4\tau = 2)$

④ c)



(i) projection of $\underline{x}_0$ onto y_1, y_2 axes is clearly $3/2$ basis vector for each axis.

$\underline{x}_0 = \frac{3}{2}\vec{b}_1 + \frac{3}{2}\vec{b}_2$

(ii) direction field arrows line up along eigenvector lines (in opposite direction showing eigenvalues are negative)