

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. **BOX** final short answers. **LABEL** parts of problem. Keep answers **EXACT** (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses.**

1.  $3x_1 + x_2 - 3x_3 + 11x_4 + 10x_5 = 0, 5x_1 + 8x_2 + 2x_3 - 2x_4 + 7x_5 = 0, 2x_1 + 5x_2 - x_4 + 14x_5 = 0$

a) Write this system in matrix form, write down the augmented matrix, use technology to evaluate its RREF form, and by hand backsubstitute to write down the general solution in matrix form, identifying the leading and free variables. Then express your final result in scalar form:  $x_1 = \dots$ , etc.

Using the work you have already done:

b) Write down a basis for the solution space of this system as a *set of column matrices*.

c) Write down the independent linear relationships among the 5 columns of the coefficient matrix. You may use the shorthand  $C_i$  to refer to the  $i$ th column of this matrix.

d) Express the last column  $\langle 10, 7, 14 \rangle$  of the coefficient matrix as a linear combination of its first four columns in the most general way (delete the final column of the augmented matrix and redo the solution). Explain what you are doing.

2. Given that  $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -2 \\ 1 & 1 & 4 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} :$

a) Using allowed row operations, evaluate by hand the determinant of  $A$ . What does this value say about the linear independence of the columns of  $A$ ?

b) Use technology to calculate the inverse of  $A$  (it may be convenient to factor out a common denominator) and then use that inverse to solve the system  $Ax = b$  for the column matrix  $x$  of unknowns, showing explicitly the hand matrix multiplication steps. Then check you solution for  $x$  by evaluating by hand the matrix product  $Ax$  and comparing it with  $b$ .

c) Write down the system of scalar equations equivalent to the matrix equation  $Ax = 2x$ , and rewrite them in the standard form for a linear homogeneous system of equations (unknowns on left hand side). Then write down the augmented matrix and its reduced row echelon form and then by hand, backsubstitute to find the general solution for the unknown column matrix  $x$ .

## ► solution

## ▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: