# MAT2705-01/06 07F TEST 1 ANSWERS

① a), b) see below

c) $\frac{dy}{dx} = -xy^2$ (separable)

$\frac{dy}{y^2} = -x\,dx$ (separate)

$\int y^{-2}dy = -\int x\,dx$ (integrate)

$-y^{-1} = -\frac{x^2}{2} + C_1$

$y^{-1} = \frac{x^2}{2} - C_1$

$y = \frac{1}{\frac{x^2}{2} - C_1} = \frac{2}{x^2 - 2C_1} = \frac{2}{x^2 + C}$

$\boxed{y = \frac{2}{x^2+C}}$ general soln

d) $\frac{3}{2} = y(1) = \frac{2}{1^2 + C}$

$C + 1 = \frac{4}{3} \rightarrow C = 4/3 - 1 = 1/3$

$\boxed{y = \frac{2}{x^2 + \frac{1}{3}} = \frac{6}{3x^2+1}}$

e) $y(0) = 6$ comparable to my estimate of 5.3 ÷ the direction field quickly levels off near the axis so it is difficult to guess/estimate where the curve should be drawn there.

(see online maple plot)

f) $\frac{d}{dx}\left(6(3x^2+1)^{-1}\right) = -x\left(\frac{6}{3x^2+1}\right)^2$

$6(-1)(3x^2+1)^{-2}(6x+0) = \frac{-x \cdot 36}{(3x^2+1)^2}$

$-\frac{36x}{(3x^2+1)^2} = -\frac{36x}{(3x^2+1)^2}$ ✓

② a) $\left[\frac{dT}{dt} + kT = 42k\right] \rightarrow e^{kt}\left[\frac{dT}{dt} + kT = 42k\right] \rightarrow \frac{d}{dt}\left(Te^{kt}\right) = 42k\,e^{kt}$

$\int k\,dt = kt$, $e^{kt}$ (integrating factor)

$Te^{kt} = \int 42k\,e^{kt}\,dt$

$= 42k\left(\frac{e^{kt}}{k}\right) + C_1$

$= 42e^{kt} + C_1$

$T = e^{-kt}(42e^{kt} + C)$

$= 42 + Ce^{-kt}$ (gen soln)

$212 = T(0) = 42 + C \rightarrow C = 212 - 42 = 170$

$\boxed{T = 42 + 170e^{-kt}}$ (IVP soln)

$91 = T(1) = 42 + 170e^{-k}$ (extra condition)

$91 - 42 = 49 = 170e^{-k}$

$e^{k} = \frac{170}{49}$

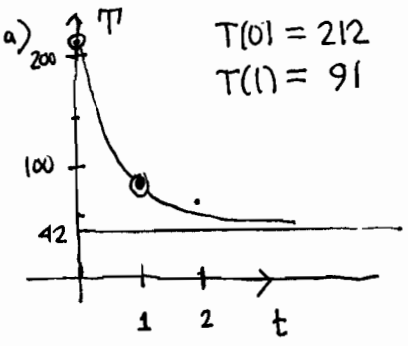
$\boxed{k = \ln\frac{170}{49} \approx 1.24398}$ keep many digits until final answer

b) $T(2) = 42 + 170e^{-2k}$

$\approx 56.1235 \approx \boxed{56°}$ ↔ 68° way off!

no way this is a good model for whatever bob did!

② a)




$T(0) = 212$

$T(1) = 91$

$\frac{dT}{dt} = -kT + 42k \rightarrow \frac{dT}{dt} + kT = 42k$

① a) b) looks like about 5.3 ≈ y(0)