MAT2705-02/03 06S Quiz 7 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology to find the integer roots of the characteristic equation.**

1. $y'' + 4y' + 148y = 0,\ y(0) = 1,\ y'(0) = 2$.

a) Find the general solution $y(x)$ by hand using the educated guess method.

b) Find the solution which satisfies the initial conditions, showing all work.

c) Rewrite your sinusoidal factor of the solution of the IVP in phase-shifted cosine form $A\cos(\omega x - \delta)$ and state the initial amplitude A and phase shift δ.

d) Use technology to plot your result of part b) for $x = 0..3$ [not part c)! so this will be a check on it], including the envelope functions $\pm A e^{-kx}$, and make a rough sketch of what you see, labeling the axes with variable names and key tickmarks on your sketch. Does the envelope you found fit the solution curve as it should? [Yes or no, with explanation would be a good response.]

Optional. Think about ***only if*** you finish early: Evaluate the period T and the characteristic length τ of your exponentially decaying sinusoidal solution function and evaluate the number of periods that fit into 4 characteristic lengths. Does this agree with your plot?

▶ solution

a) $y'' + 4y' + 148y = 0 \xrightarrow{y=e^{rx}} (r^2 + 4r + 148)e^{rx} = 0 \rightarrow r^2 + 4r + 148 = 0$

$r = \frac{-4 \pm \sqrt{16 - 4(148)}}{2} = \frac{-4 \pm \sqrt{16 - 4\cdot 4\cdot 37}}{2} = \frac{-4 \pm 4\sqrt{1-37}}{2} = -2 \pm 2\sqrt{-36} = -2 \pm 12i$

$e^{rx} = e^{-2x} e^{\pm 12ix} \rightarrow e^{-2x}\cos 12x,\ e^{-2x}\sin 12x \rightarrow \boxed{y = e^{-2x}(c_1 \cos 12x + c_2 \sin 12x)}$

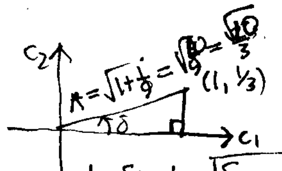
b) $y' = -2e^{-2x}(c_1\cos 12x + c_2 \sin 12x) + e^{-2x}(-12c_1 \sin 12x + 12c_2 \cos 12x)$

$y(0) = c_1 = 1$

$y'(0) = -2c_1 + 12c_2 = 2$

$\rightarrow c_2 = \frac{2+2c_1}{12} = \frac{4}{12} = \frac{1}{3} \rightarrow \boxed{y = e^{-2x}(\cos 12x + \frac{1}{3}\sin 12x)}$

c)

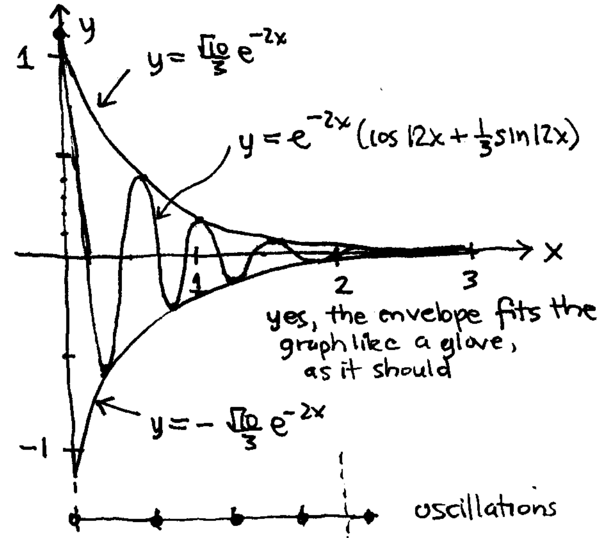


$\boxed{A = \frac{\sqrt{10}}{3}} \approx 1.054$

$\tan\delta = 1/3 \rightarrow \boxed{\delta = \arctan 1/3} \approx 0.3218 \approx 18.4°$

$$\boxed{y = \underbrace{\frac{\sqrt{10}}{3} e^{-2x}}_{A(x)} \cos(12x - \arctan 1/3)}$$

d)



OPTIONAL COMMENT

$\tau = \frac{1}{2} \quad \omega = 12 \rightarrow T = \frac{2\pi}{12} \approx 0.52$

$\frac{4\tau}{T} = \frac{4(\frac{1}{2})}{2\pi/12} = \frac{12}{\pi} \approx 3.82$ nearly 4 oscillations → yes, the plot agrees