MAT2705-02/03 06S Quiz 6 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $y''+3\,y'+2\,y=0,\ y(0)=1,\ y'(0)=2$.

a) Find the general solution by hand using the educated guess method.

b) Find the solution which satisfies the initial conditions, showing all work.

c) Use technology to make plot your result for $x=0\,..4$ and make a rough sketch of what you see, labeling the axes with variable names and key tickmarks on your sketch.

d) Use calculus to determine exactly the x and y values of the obvious peak and then their approximate values, but if you get stuck on solving the derivative condition exactly, use technology to find the approximate values in any way you can. Do the numbers you found agree with what your eyes see in the technology plot? [Yes or no, with explanation would be a good response.]

Optional. Think about ***only if*** you finish early: considering the characteristic lengths of the exponentials involved, why is this plot range a good choice?

▶ solution

a) $y=e^{rx} \to y''+3y'+2y=0$

$$r^2e^{rx}+3re^{rx}+2e^{rx}=0$$

$$(r^2+3r+2)e^{rx}=0$$

$$r^2+3r+2=0$$

$$r=\frac{-3\pm\sqrt{9-4(2)(1)}}{2}=\frac{-3\pm1}{2}=-1,-2$$

$$e^{rx}=e^{-x},e^{-2x}$$

$$\boxed{y=c_1e^{-x}+c_2e^{-2x}}$$

b) $y'=-c_1e^{-x}-2c_2e^{-2x}$

$$1=y(0)=c_1+c_2$$

$$2=y'(0)=-c_1-2c_2$$

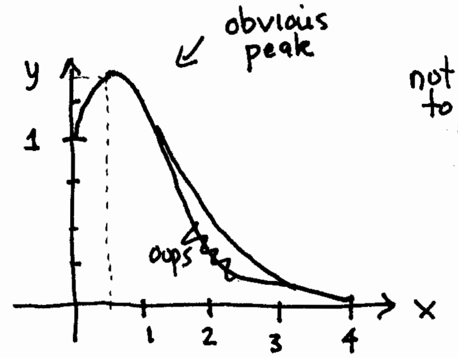
$$\begin{bmatrix}1&1\\-1&-2\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$$ row reduce or:

$$\begin{bmatrix}c_1\\c_2\end{bmatrix}=\begin{bmatrix}1&1\\-1&-2\end{bmatrix}^{-1}\begin{bmatrix}1\\2\end{bmatrix}=\frac{1}{-2+1}\begin{bmatrix}-2&-1\\1&1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$=\begin{bmatrix}2&1\\-1&-1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}4\\-3\end{bmatrix}$$

$$\boxed{y=4e^{-x}-3e^{-2x}}$$

c)



not so easy to draw what you see, is it?

looks right! yes!

d) $y=4e^{-x}-3e^{-2x}$

$$y'=4(-e^{-x})-3(-2e^{-2x})$$

$$=-4e^{-x}+6e^{-2x}=0$$

$$[4e^{-x}=6e^{-2x}]\,e^{2x}/4$$

$$e^{x}=\frac{6}{4}=\frac{3}{2}$$

$$\boxed{x=\ln 3/2 \approx 0.405}$$

$$y=4e^{-\ln\frac{3}{2}}-3e^{-2\ln\frac{3}{2}}$$

$$=4(e^{\ln 3/2})^{-1}-3(e^{\ln\frac{3}{2}})^{-2}$$

$$=4\left(\tfrac{3}{2}\right)^{-1}-3\left(\tfrac{3}{2}\right)^{-2}$$

$$=4\cdot\left(\tfrac{2}{3}\right)-3\left(\tfrac{4}{9}\right)=\frac{8}{3}-\frac{4}{3}=\boxed{\frac{4}{3}\approx 1.33}$$

$y=$

optional

$$y=4e^{-x}-3e^{-2x}$$

characteristic length: 1 $\qquad$ $1/2$ (decays faster)

$x=0..4$ is 4 characteristic lengths of the slowest decaying exponential so it should show all the interesting behavior