MAT2705-02/03 06S Quiz 5 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).
[You may use technology for row reductions, backsubstitutions and determinants although these may easily be done by hand.]

1. $v1 = \langle 1,0,1,0\rangle$, $v2 = \langle 0,1,0,1\rangle$, $v3 = \langle 1,-1,1,-1\rangle$, $v4 = \langle 1,2,1,2\rangle$, $v5 = \langle 2,1,2,1\rangle$.
a) Express $v5$ as a linear combination of the remaining 4 vectors. [Final answer: $v5 = \ldots$]
b) Check that this linear combination that you find actually evaluates to $v5$.
b) Find the independent linear relationships among these 4 vectors. Write out these relationships.
2. Demonstrate that the following vectors are linearly independent: (OPTIONAL)
$u1 = \langle 1,-1,0,0\rangle$, $u2 = \langle 1,1,-1,-1\rangle$, $u3 = \langle 0,0,1,1\rangle$, $u4 = \langle 1,-1,1,-1\rangle$.

▶ solution

1.a) $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_4 = \vec{v}_5$

$$\begin{bmatrix}1&0&1&1\\0&1&-1&2\\1&0&1&1\\0&1&-1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}2\\1\\2\\1\end{bmatrix} \xrightarrow{\text{augment}} \begin{bmatrix}1&0&1&1&2\\0&1&-1&2&1\\1&0&1&1&2\\0&1&-1&2&1\end{bmatrix} \xrightarrow{R_3\to R_3-R_1} \begin{bmatrix}1&0&1&1&2\\0&1&-1&2&1\\0&0&0&0&0\\0&1&-1&2&1\end{bmatrix} \xrightarrow{R_4\to R_4-R_2} \begin{array}{c}\begin{bmatrix}1&0&1&1&2\\0&1&-1&2&1\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\\ \begin{matrix}L&L&F&F\\x_1&x_2&x_3&x_4\end{matrix}\end{array}$$

$x_3=t_1$, $x_4=t_2$

$$\begin{aligned}x_1+t_1+t_2&=2\\x_2-t_1+2t_2&=1\\0&=0\\0&=0\end{aligned} \quad\rightarrow\quad \boxed{\begin{aligned}x_1&=2-t_1-t_2\\x_2&=1+t_1-2t_2\\x_3&=t_1\\x_4&=t_2\end{aligned}} \text{ or } \boxed{\vec{x}=\begin{bmatrix}2-t_1-t_2\\1+t_1-2t_2\\t_1\\t_2\end{bmatrix}}$$

[setting $t_1=0=t_2$ would have given you one particular way of doing this]

so $\vec{v}_5 = (2-t_1-t_2)\vec{v}_1 + (1+t_1-2t_2)\vec{v}_2 + t_1\vec{v}_3 + t_2\vec{v}_4$

b)
$$= (2-t_1-t_2)\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + (1+t_1-2t_2)\begin{bmatrix}0\\1\\0\\1\end{bmatrix} + t_1\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix} + t_2\begin{bmatrix}1\\2\\1\\2\end{bmatrix} = \begin{bmatrix}(2\cancel{-t_1-t_2})\cancel{+t_1+t_2}\\0+(1\cancel{+t_1-2t_2})\cancel{-t_1+2t_2}\\(2\cancel{-t_1-t_2})\cancel{+t_1+t_2}\\0+(1\cancel{+t_1-2t_2})\cancel{-t_1+2t_2}\end{bmatrix} = \begin{bmatrix}2\\1\\2\\1\end{bmatrix} \checkmark$$

c) Solve homogeneous linear system instead:

$$x_1\vec{v}_1+x_2\vec{v}_2+x_3\vec{v}_3+x_4\vec{v}_4=0 \longrightarrow \vec{x}=\begin{bmatrix}-t_1-t_2\\t_1-2t_2\\t_1\\t_2\end{bmatrix} = t_1\begin{bmatrix}-1\\1\\1\\0\end{bmatrix} + t_2\begin{bmatrix}-1\\-2\\0\\1\end{bmatrix}$$

coeffs of 2 independent relationships

$$\boxed{\begin{aligned}-\vec{v}_1+\vec{v}_2+\vec{v}_3&=0\\-\vec{v}_1-2\vec{v}_2+\vec{v}_4&=0\end{aligned}}$$

(check: $-\begin{bmatrix}1\\0\\1\\0\end{bmatrix}+\begin{bmatrix}0\\1\\0\\1\end{bmatrix}+\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix}=\begin{bmatrix}-1+1\\1-1\\-1+1\\1-1\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}\checkmark$

$-\begin{bmatrix}1\\0\\1\\0\end{bmatrix}-2\begin{bmatrix}0\\1\\0\\1\end{bmatrix}+\begin{bmatrix}1\\2\\1\\2\end{bmatrix}=\begin{bmatrix}-1+1\\-2+2\\-1+1\\-2+2\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}\checkmark$)

2. $\underline{U} = \langle u_1|u_2|u_3|u_4\rangle = \begin{bmatrix}1&1&0&1\\-1&1&0&-1\\0&-1&1&1\\0&-1&1&-1\end{bmatrix} \xrightarrow{R_2\to R_2+R_1} \begin{bmatrix}1&1&0&1\\0&2&0&0\\0&-1&1&1\\0&-1&1&-1\end{bmatrix} \xrightarrow[R_4\to R_4+\frac12 R_2]{R_3\to R_3+\frac12 R_2} \begin{bmatrix}1&1&0&1\\0&2&0&0\\0&0&1&1\\0&0&1&-1\end{bmatrix} \xrightarrow{R_4\to R_4-R_3} \begin{bmatrix}1&1&0&1\\0&2&0&0\\0&0&1&1\\0&0&0&-2\end{bmatrix}$

$\det \underline{U} = 1\cdot2\cdot1(-2) = -4$ or evaluate using technology. nonzero determinant means they are linearly independent, equivalent to U row reducing to the identity [$R_2\to \frac12 R_2$, $R_4\to -\frac12 R_4$ followed by add row operations to eliminate entries above leading 1's]
GaussJordan row reduction by technology reduces U to identity, so solution of $U\vec{x}=\vec{0}$ reduces to $\vec{x}=\vec{0}$, ie, no linear relationships exist among these vectors.