MAT2705-02/03 06S Quiz 4 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

$$2\,x_1 + 4\,x_2 = 6$$
$$3\,x_1 + 5\,x_2 = 7$$

a) Write down the coefficient matrix **A**, the RHS matrix **b** and the augmented matrix **C** = <**A** | **b**> for this linear system of equations.

b) Evaluate the inverse matrix A^{-1} by hand in 4 easy steps, annotating the MultiplyRow, AddRow, SwapRow operations you apply to each successive matrix in the process, using your own words or the notation: $R_1 \to 3\ R_1, R_1 \to R_1 + 2\ R_1,\ R_1 \leftrightarrow R_2$.

c) Check that your inverse matrix actually satisfies the condition $A^{-1}A = I$ by showing the hand multiplication steps explicitly.

d) Use the inverse matrix to solve the original system of equations, writing the solution first in (column matrix) vector form ($\mathbf{x} =$ ...) and then in scalar form ($x_1 = ..., x_2 = ...$).

e) Check by back substitution of your solutions for the unknowns in the original equations that your solutions actually solve that system of equations.

▶ solution

a) $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \quad b = \begin{bmatrix} 6 \\ 7 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

b) $\langle A \mid I_2 \rangle = \begin{bmatrix} 2 & 4 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{bmatrix} \xrightarrow[R_1 \to \frac{1}{2}R_1]{} \begin{bmatrix} 1 & 2 & 1/2 & 0 \\ 3 & 5 & 0 & 1 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 3R_1]{} \begin{bmatrix} 1 & 2 & 1/2 & 0 \\ 0 & -1 & -3/2 & 1 \end{bmatrix}$

$-3 \;\; -6 \;\; -3/2 \;\; -0$

$-0 \;\; -2 \;\; -3 \;\; +2$

$\xrightarrow[R_2 \to -R_2]{} \begin{bmatrix} 1 & 2 & 1/2 & 0 \\ 0 & 1 & 3/2 & -1 \end{bmatrix} \xrightarrow[R_1 \to R_1 - 2R_2]{} \begin{bmatrix} 1 & 0 & -5/2 & 2 \\ 0 & 1 & 3/2 & -1 \end{bmatrix}$

$\boxed{A^{-1} = \begin{bmatrix} -5/2 & 2 \\ 3/2 & -1 \end{bmatrix}} = \frac{1}{2}\begin{bmatrix} -5 & 4 \\ 3 & -2 \end{bmatrix}$

c) $A^{-1}A = \frac{1}{2}\begin{bmatrix} -5 & 4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} -5(2)+4(3) & -5(4)+4(5) \\ 3(2)-2(3) & 3(4)-2(5) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

oops I almost added instead of multiplied! $5(6) \neq 11$!

d) $A^{-1}[Ax = b] \to x = A^{-1}b = \frac{1}{2}\begin{bmatrix} -5 & 4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -5(6)+4(7) \\ 3(6)-2(7) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 28-30 \\ 18-14 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\boxed{x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \text{or} \quad x_1 = -1, x_2 = 2}$

e) $2(-1) + 4(2) = -2 + 8 = 6$ ✓

$3(-1) + 5(2) = -3 + 10 = 7$ ✓