MAT2705-02/03 06S Quiz 2 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

$$y' = \frac{1 - 2\,x\,y}{1 + x^2}\ , \quad y(0) = -1$$

a) Using the linear DE algorithm, put this DE into standard form and step by step derive its general solution.
b) Find the particular solution which satisfies the initial condition.
c) Show by backsubstitution (everywhere) into the DE that your particular solution actually does solve the DE.

▶ solution

a) $y' = \frac{1}{1+x^2} - \frac{2x}{1+x^2}y$

$(1+x^2)\left[y' + \frac{2x}{1+x^2}y = \frac{1}{1+x^2}\right] \rightarrow \frac{d}{dx}\left(y(1+x^2)\right) = \frac{1+x^2}{1+x^2} = 1$

$y(1+x^2) = \int 1\,dx = x + C$

$\int \frac{2x}{1+x^2}dx = \int \frac{dy}{u} = \ln|u| = \ln(1+x^2)$

$e^{\ln(1+x^2)} = 1+x^2$

$$\boxed{y = \frac{x+C}{1+x^2}}$$ general soln

b) $-1 = y(0) = \frac{0+C}{1+0^2} = C \rightarrow C = -1 \rightarrow$ $$\boxed{y = \frac{x-1}{1+x^2}}$$ soln of IVP = DE + I.C

not end of this part b) → "Find the particular solution..." means deliver a complete expression for y(x) as the response to this instruction

c) $y' = \frac{1-2xy}{1+x^2}$ ← backsubstitute for y everywhere in DE, don't manipulate equation, only simplify separately LHS & RHS see if they are equal

$$\left(\frac{x-1}{1+x^2}\right)' \stackrel{?}{=} \frac{1-2x\left(\frac{x-1}{1+x^2}\right)}{1+x^2}$$

$$\frac{(1+x^2)(1)-(x-1)(2x)}{(1+x^2)^2} \qquad \frac{1+x^2-2x(x-1)}{(1+x^2)^2}$$

$$\frac{1+x^2-2x^2+2x}{(1+x^2)^2} \qquad \frac{1+x^2-2x^2+2x}{(1+x^2)^2}$$

$$\frac{1+2x-x^2}{(1+x^2)^2} \stackrel{\checkmark}{=} \frac{1+2x-x^2}{(1+x^2)^2}$$

yes, this is a soln.