

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). **You may use technology for row reductions and root finding.** You are encouraged to use technology to check all of your hand results.

1.  $x'' + 20x' + 64x = 32 \cdot 41 \cos(20t)$ ,  $x(0) = 0$ ,  $x'(0) = 64$ .

- Find the general solution by hand, showing all steps.
- Find the initial value problem solution by hand, showing all steps.
- What are the characteristic times for the exponential terms in your solution? What are the periods for the sinusoidal terms in your solution? Explain.
- Optional:** Make a technology plot of the ivp solution together with the steady state periodic solution in a window that shows all behavior appropriately and make a rough sketch of what you see, completely labeled with tickmarks, axis labels, etc. Explain how you choose the time interval of the window.
- Optional:** Is the amplitude of the steady periodic solution that you see in your plot consistent with what you can calculate from its expression? Explain.

2.  $x_1' = -3x_1 + 2x_3$ ,  $x_2' = 3x_1 - 10x_2$ ,  $x_3' = 10x_2 - 2x_3$ ,  
 $x_1(0) = 0$ ,  $x_2(0) = 0$ ,  $x_3(0) = 28$ .

- Rewrite this system of DEs and its initial conditions in matrix form for the vector variable  $\mathbf{x} = \langle x_1, x_2, x_3 \rangle$ .
- Find the general solution, by hand, showing all steps.
- Find the solution which satisfies the initial conditions, by hand, showing all steps.
- What are the limiting values of these variables for large  $t$ , determined from your formulas?
- Optional:** Make a technology plot which captures the all the behavior of this solution for  $t > 0$ , and sketch the result on your paper with appropriate labels and tickmarks. Do your calculated values correspond to what you see in the plot?

3.  $x_1'' = -12x_1 + 2x_2$ ,  $x_2'' = 16x_1 - 8x_2$ ,  $x_1(0) = 1$ ,  $x_2(0) = 1$ ,  $x_1'(0) = 0$ ,  $x_2'(0) = 0$ .

- Rewrite this system of DEs and its initial conditions in matrix form for the vector variable  $\mathbf{x} = \langle x_1, x_2 \rangle$ .
- Find the general solution by hand, showing all steps.
- Find the solution which satisfies the initial conditions, by hand, showing all steps.
- This has two modes: a "tandem" mode (same sign values of the two unknowns) and an "accordion" mode (opposite sign values of the two unknowns). What are the frequencies and what are the relative amplitudes (their ratios) of the variables  $x_1$  and  $x_2$  in each of these oscillating modes?
- Optional:** Make a technology plot of your solution functions versus the time for exactly two periods of the motion, and sketch the result on your paper with appropriate labels and tickmarks. Do your curves repeat exactly twice in the plot as they should?

## ► solution

## ▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:  
 "During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: \_\_\_\_\_

Date: \_\_\_\_\_