MAT2705-02/03 06S Take Home Test 3 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). **You may use technology for row reductions and root finding.** Print requested technology plots, annotate them appropriately and attach to the relevant problems.

1. The charge $q(t)$ in coulombs on a capacitor in an RLC circuit is described by

$$L\,q''(t) + R\,q'(t) + C^{-1}\,q(t) = E(t).$$

Let $L = 2$ henries, $R = 4$ ohms, $C = 0.05$ farads. Consider the initial conditions $q(0) = 0$, $q'(0) = 0$ and the following driving voltage functions $E(t)$ in volts:

a) $E(t) = 100$.
Find the initial value problem solution by hand and evaluate the asymptotic value $q_\infty = \lim_{t\to\infty} q(t)$. Make a single plot in an appropriate viewing window showing both the solution function and its horizontal asymptote.

b) $E(t) = 100\,e^{-\frac{t}{10}}$.
Find the initial value problem solution by hand and evaluate the asymptotic value $q_\infty = \lim_{t\to\infty} q(t)$. Make a single plot in an appropriate viewing window showing both the solution function and its horizontal asymptote.

c) $E(t) = 100\cos(3\,t)$.
Find the initial value problem solution by hand and evaluate the steady state solution value [of the amplitude. (*deleted by mistake: ignore*)] Make a single plot in an appropriate viewing window showing both the solution function and the steady state solution.

d) What are the natural frequency ω_0, natural decay time τ_0, and the quality factor $Q = \omega_0\tau_0$ (both exact and numeric values) for this RLC circuit?

e) $E(t) = 100\cos(\omega\,t)$.
Explore resonance for this circuit by finding the steady state solution by hand, where the nonegative frequency ω of the driving voltage function is a parameter.

f) Evaluate the steady state amplitude function $A(\omega)$ and use calculus to find the exact and numerical value of the frequency ω_p and amplitude where it has its peak value.
What is the numerical value of the ratio $A(\omega_p)/A(0)$? How does this compare to Q?

g) Plot this amplitude function in an appropriate window together with the constant functions $A(\omega_0)$, $A(\omega_p)$ and $A(0)$. Is this consistent with what you have found?

2. $x_1'(t) = -2\,x_1(t) + 9\,x_3(t)$, $x_2'(t) = 2\,x_1(t) - 2\,x_2(t)$, $x_3'(t) = 2\,x_2(t) - 9\,x_3(t)$,
$x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 1$.

a) Write this system and its initial conditions in matrix form, i.e., for the vector variable $\mathbf{x} = \langle x_1, x_2, x_3\rangle$.

b) Use the eigenvector approach to find its general solution, showing all steps.

c) Find the IVP solution, showing all steps.

d) Identify the limiting constant solution $\mathbf{x}_\infty = \lim_{t\to\infty} \mathbf{x} = \langle x_{\infty 1}, x_{\infty 2}, x_{\infty 3}\rangle$ that it approaches asymptotically for large t.

e) Make a single plot showing the three solution curves versus t on the same axes together with their horizontal asymptotes found in d) for at least 5 characteristic times of the slowest decaying exponential term in these expressions (i.e., in an appropriate viewing window).

f) Use calculus to find the local extrema for each of these three functions for $t > 0$ numerically giving both t and x_i to 3 significant digits. Are your results consistent with your plot?

g) Find the values of t where each of these functions reaches within one percent of its asymptotic value (i.e., 1.01 or 0.99 times the corresponding asymptotic value). Do these values seem consistent with your plot? Explain.

3. $x_1'(t) = x_1(t) - \frac{3}{2}\, x_2(t),\ x_2'(t) = 3\, x_1(t) - 2\, x_2(t),\ x_1(0) = 1,\ x_2(0) = 1$.

a) Write this system and its initial conditions in matrix form, i.e., for the vector variable $\mathbf{x} = \langle x_1, x_2 \rangle$.
b) Use the eigenvector approach to find its general solution, showing all steps.
c) Find the IVP solution, showing all steps.
d) Express the sinusoidal factor in each solution function as a phase-shifted cosine.
e) Make a single $t > 0$ plot for an interval 5 times the characteristic time of the exponential factor, showing both solution curves and their exponentially decaying amplitude envelopes of the oscillations, annotating your paper printout to identify the individual solution curves by name. Estimate the time interval between successive peaks of these two damped oscillations. [clarification: successive peaks on different curves]
f) What is the phase shift between these peaks in radians, degrees and in a fraction of a cycle? Are these consistent with your estimate for the time interval between successive peaks in e)? Explain. Which of the two is ahead in time (has its peaks first)?

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have read the long instructions on the class web page. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: