Mat 2705-02/03 06S Test 2 Answers

① a) $\begin{bmatrix}3\\2\\-2\end{bmatrix} = x_1\begin{bmatrix}1\\0\\0\end{bmatrix} + x_2\begin{bmatrix}1\\1\\1\end{bmatrix} + x_3\begin{bmatrix}1\\2\\4\end{bmatrix}$

$$\begin{bmatrix}1&1&1\\0&1&2\\0&1&4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}3\\2\\-2\end{bmatrix} \to \begin{bmatrix}1&1&1&3\\0&1&2&2\\0&1&4&-2\end{bmatrix} \xrightarrow[R_1\to R_1-R_2]{R_3\to R_3-R_2}$$

$$\begin{bmatrix}1&0&-1&1\\0&1&2&2\\0&0&2&-4\end{bmatrix} \xrightarrow[\frac{1}{2}R_3]{R_3 \to} \begin{bmatrix}1&0&-1&1\\0&1&2&2\\0&0&1&-2\end{bmatrix} \xrightarrow[R_2\to R_2-2R_3]{R_1\to R_1+R_3}$$

$$\begin{bmatrix}1&0&0&-1\\0&1&0&6\\0&0&1&-2\end{bmatrix} \qquad \begin{aligned}x_1&=-1\\x_2&=6\\x_3&=-2\end{aligned}$$

so $\boxed{\begin{bmatrix}3\\2\\-2\end{bmatrix} = -\begin{bmatrix}1\\0\\0\end{bmatrix} + 6\begin{bmatrix}1\\1\\1\end{bmatrix} - 2\begin{bmatrix}1\\2\\4\end{bmatrix}}$

b) $= \begin{bmatrix}-1+6-2\\0+6-4\\0+6-8\end{bmatrix} = \begin{bmatrix}3\\2\\-2\end{bmatrix}$ ✓

② $y''' - 3y'' + 2y' = 0$

a) $y = e^{rx} \nearrow \quad (r^3 - 3r^2 + 2r)\, e^{rx} = 0$

$r(r^2 - 3r + 2) = 0$

$r(r-1)(r-2) = 0$

$r = 0, 1, 2$

$e^{rx} = \underbrace{e^{0x}}_{1}, e^x, e^{2x}$ basis of soln space

$\boxed{y = c_1 + c_2 e^x + c_3 e^{2x}}$ general soln

b) $y' = \quad c_2 e^x + 2c_3 e^{2x}$

$y'' = \quad c_2 e^x + 4c_3 e^{2x}$

$y(0) = c_1 + c_2 + c_3 = 3$

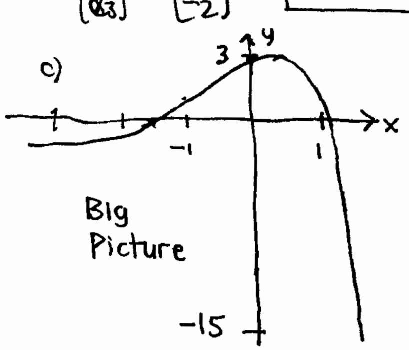
$y'(0) = \quad c_2 + 2c_3 = 2$

$y''(0) = \quad c_2 + 4c_3 = -2$

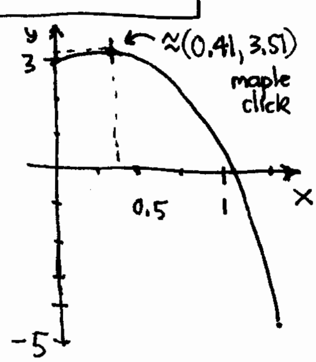
$\begin{bmatrix}1&1&1\\0&1&2\\0&1&4\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}3\\2\\-2\end{bmatrix}$ same as problem ①

$\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}-1\\6\\-2\end{bmatrix} \to \boxed{y = -1 + 6e^x - 2e^{2x}}$

c)



Big Picture



$y = -1 + 6e^x - 2e^{2x}$

$y' = \quad 6e^x - 4e^{2x} = 0$

$6e^x = 4e^{2x}$

$\frac{3}{2} = e^x \quad \boxed{x = \ln\frac{3}{2} \approx 0.4055}$

$y = -1 + 6e^{\ln\frac{3}{2}} - 2e^{2\ln\frac{3}{2}}$

$= -1 + 6\left(\frac{3}{2}\right) - 2\left(\frac{3}{2}\right)^2$

$= -1 + 9 - \frac{9}{2} = 8 - \frac{9}{2} = \boxed{\frac{7}{2} = 3.5 = y}$

These numbers are dead on.

③ a) $W(x) = \begin{bmatrix}\cos x & \cos 3x & \cos^3 x\\ -\sin x & -3\sin 3x & -3\cos^2 x \sin x\\ -\cos x & -9\cos 3x & +6\cos x \sin^2 x - 3\cos^3 x\end{bmatrix}$

$W(0) = \begin{bmatrix}1&1&1\\0&0&0\\-1&-9&-3\end{bmatrix} = A$

b) $Ax = 0: \left[\begin{array}{ccc|c}1&1&1&0\\0&0&0&0\\-1&-9&-3&0\end{array}\right] \xrightarrow[R_3+R_1]{R_3\to} \begin{bmatrix}1&1&1&0\\0&0&0&0\\0&-8&-2&0\end{bmatrix}$

$\xrightarrow[R_2\to -\frac{1}{8}R_2]{R_2 \leftrightarrow R_3} \begin{bmatrix}1&1&1&0\\0&1&\frac{1}{4}&0\\0&0&0&0\end{bmatrix} \xrightarrow{R_1\to R_1-R_2} \begin{array}{cccc}L&L&F&\\ \hline 1&0&3/4&0\\0&1&1/4&0\\0&0&0&0\end{array} \begin{aligned}&\to x_1 = -3/4\, x_3\\&\to x_2 = -1/4\, x_3\\&\quad x_3 = t\end{aligned}$

$x_1 = -\frac{3}{4}t, \; x_2 = -\frac{1}{4}t, \; x_3 = t$ so

$-\frac{3}{4}t \cos x - \frac{1}{4}t\cos 3x + t\cos^3 x = 0$ or

$\boxed{-3\cos x - \cos 3x + 4\cos^3 x = 0}$

solve for $\cos 3x$:

c) $\boxed{\cos 3x = 4\cos^3 x - 3\cos x}$

d) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \cos^3\frac{\pi}{6} = \left(\frac{3^{1/2}}{2}\right)^3 = \frac{3\sqrt{3}}{8}$

1, $\pi/6$, $\sqrt{3}/2$, ½

$\cos 3\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{2} = 0$

$\cos 3\left(\frac{\pi}{6}\right) \overset{?}{=} 4\cos^3\frac{\pi}{6} - 3\cos\frac{\pi}{6}$

$\overset{\shortparallel}{0} \qquad \frac{4 \cdot 3\sqrt{3}}{8_2} - 3\frac{\sqrt{3}}{2} = 0$ ✓

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