Quiz 9 Not! 055 Mat2705

a) $\begin{bmatrix} y_1'' \\ y_2'' \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -6 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\begin{bmatrix} -y_1 \\ -6y_2 \end{bmatrix}} + \underbrace{\begin{bmatrix} 1/5 & 2/5 \\ -2/5 & 1/5 \end{bmatrix}\begin{bmatrix} 0 \\ A_0 \sin 2t \end{bmatrix}}_{\begin{bmatrix} \frac{2}{5}A_0 \sin 2t \\ \frac{1}{5}A_0 \sin 2t \end{bmatrix}} = \begin{bmatrix} -y_1 + \frac{2A_0}{5}\sin 2t \\ -6y_2 + \frac{A_0}{5}\sin 2t \end{bmatrix}$ → $y_1'' = -y_1 + \frac{2A_0}{5}\sin 2t$, $y_2'' = -6y_2 + \frac{A_0}{5}\sin 2t$

$$\boxed{\begin{aligned} y_1'' + y_1 &= \tfrac{2A_0}{5}\sin 2t \\ y_2'' + 6y_2 &= \tfrac{A_0}{5}\sin 2t \end{aligned}}$$

b) $A_0 = 0$. $y_1'' + y_1 = 0$

$y_1 = e^{rt}$ ↘ $r^2 + 1 = 0$, $r = \pm i$

$y_1 = e^{\pm it} = \cos t \pm i \sin t$ → $\cos t, \sin t$

$y_1 = c_1 \cos t + c_2 \sin t$

$y_2'' + 6y_2 = 0$

$y_2 = e^{rt}$ ↘ $r^2 + 6 = 0$, $r = \pm\sqrt{6}\,i$

$y_2 = e^{\pm\sqrt{6}\,i} = \cos\sqrt{6}t \pm i\sin\sqrt{6}t$ → $\cos\sqrt{6}t, \sin\sqrt{6}t$

$y_2 = c_3 \cos\sqrt{6}t + c_4 \sin\sqrt{6}t$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} c_1\cos t + c_2 \sin t \\ c_3 \cos\sqrt{6}t + c_4 \sin\sqrt{6}t \end{bmatrix}$

$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ → $\begin{bmatrix} c_1 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1/5 & 2/5 \\ -2/5 & 1/5 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -c_1\sin t + c_2\cos t \\ -\sqrt{6}c_3\sin\sqrt{6}t + \sqrt{6}c_4\cos\sqrt{6}t \end{bmatrix}$

$\begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} c_2 \\ \sqrt{6}c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ $\begin{bmatrix} c_2 \\ \sqrt{6}c_4 \end{bmatrix} = \begin{bmatrix} 1/5 & 2/5 \\ -2/5 & 1/5 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -2/5 \\ -1/5 \end{bmatrix}$

$c_1 = 0$, $c_2 = -2/5$, $c_3 = 0$, $c_4 = -\frac{1}{5\sqrt{6}} = -\frac{\sqrt{6}}{30}$

$y_1 = -\frac{2}{5}\sin t$

$y_2 = -\frac{1}{5\sqrt{6}}\sin\sqrt{6}t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -\frac{2}{5}\sin t \\ -\frac{1}{5\sqrt{6}}\sin\sqrt{6}t \end{bmatrix} = \begin{bmatrix} -\frac{2}{5}\sin t + \frac{2}{5\sqrt{6}}\sin\sqrt{6}t \\ -\frac{4}{5}\sin t - \frac{1}{5\sqrt{6}}\sin\sqrt{6}t \end{bmatrix}$$

$\frac{\sqrt{6}}{15}$, $-\frac{\sqrt{6}}{30}$

$\max(x_1) < \frac{2}{5} + \frac{2}{5\sqrt{6}} \approx 0.563$

$\max(x_2) < \frac{4}{5} + \frac{1}{5\sqrt{6}} \approx 0.882$

(when both sines are near their extreme values of the same sign together)

c) $y_1'' + y_1 = \frac{2A_0}{5}\sin 2t$

$y_1 = c_1\cos 2t + c_2 \sin 2t$

$y_1'' = -4c_1\cos 2t - 4c_2\sin 2t$

$y_1'' + y_1 = \underbrace{-3c_1}_{0}\cos 2t - \underbrace{3c_2}_{\frac{2A_0}{5}}\sin 2t = \frac{2A_0}{5}\sin 2t$

$c_1 = 0$, $c_2 = -\frac{2A_0}{15}$

$y_1 = -\frac{2A_0}{15}\sin 2t$

$y_2'' + 6y_2 = \frac{A_0}{5}\sin 2t$

$y_2 = c_3\cos 2t + c_4\sin 2t$

$y_2'' = -4c_3\cos 2t - 4c_4\sin 2t$

$y_2'' + 6y_2 = \underbrace{2c_3}_{0}\cos 2t + \underbrace{2c_4}_{=\frac{A_0}{5}}\sin 2t = \frac{A_0}{5}\sin 2t$

$c_3 = 0$, $c_4 = \frac{A_0}{10}$

$y_2 = \frac{A_0}{10}\sin 2t$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -\frac{2}{15}A_0\sin 2t \\ \frac{1}{10}A_0\sin 2t \end{bmatrix} = \begin{bmatrix} -\frac{2}{15} + \frac{1}{5} \\ -\frac{4}{15} + \frac{1}{10} \end{bmatrix} A_0\sin 2t = A_0\sin 2t\begin{bmatrix} -1/3 \\ -1/6 \end{bmatrix} = \begin{bmatrix} -\frac{A_0}{3}\sin 2t \\ -\frac{A_0}{6}\sin 2t \end{bmatrix}$

I mess up too.
The key is to be able to track down errors.