MAT2705-01/04 05S Quiz 7 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

IVP: $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 148y = 145\cos 12t$, $y(0) = 0 = y'(0)$.

a) The roots of the characteristic equation for the related homogeneous DEQ are $r = -2 \pm 12i$. Write down the general homogeneous solution y_h.

b) Starting with the trial particular function $y_p = C_3\cos 12t + C_4 \sin 12t$, backsubstitute into the above DEQ and derive 2 linear equations needed to determine C_3 and C_4. Box it.

c) Write down the augmented matrix for that linear system and use technology to rref it and then write down the result and from it the solution of the system and then finally give your result for y_p [the correct result is $y_p = \frac{1}{4}\cos 12t + 3\sin 12t$].

d) Now impose the initial conditions on the complete solution $y = y_h + y_p$ to solve the initial value problem, quoting your final answer.

a) $\boxed{y_h = e^{-2t}(C_1\cos 12t + C_2 \sin 12t)}$

b) $148[y_p = C_3\cos 12t + C_4\sin 12t]$
$4[y_p' = -12C_3\sin 12t + 12C_4\cos 12t]$
$1[y_p'' = -144C_3\cos 12t - 144C_4\sin 12t]$

$y_p'' + 4y_p' + 148y_p = [(148-144)C_3 + 48C_4]\cos 12t$
$+ [-48C_3 + (148-144)C_4]\sin 12t$
$= \underbrace{(4C_3 + 48C_4)}_{145}\cos 12t + \underbrace{(-48C_3 + 4C_4)}_{0}\sin 12t = 145\cos 12t$

$\boxed{\begin{aligned} 4C_3 + 48C_4 &= 145 \\ -48C_3 + 4C_4 &= 0 \end{aligned}}$

c) $\begin{bmatrix} 4 & 48 & 145 \\ -48 & 4 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 1/4 \\ 0 & 1 & 3 \end{bmatrix}$

$C_3 = 1/4$, $C_4 = 3$

$\boxed{y_p = \frac{1}{4}\cos 12t + 3\sin 12t}$

d) $y = e^{-2t}(C_1\cos 12t + C_2\sin 12t) + \frac{1}{4}\cos 12t + 3\sin 12t$

$y' = -2e^{-2t}(C_1\cos 12t + C_2\sin 12t) - \frac{1}{4}\sin 12t + 3\cdot 12\cos 12t$
$+ e^{-2t}(-12C_1\sin 12t + 12C_2\cos 12t)$

$y(0) = C_1 + \frac{1}{4} = 0 \rightarrow C_1 = -1/4$

$y'(0) = -2C_1 + 12C_2 + 36 = 0 \rightarrow 12C_2 = 2(-1/4) - 36 = -\frac{73}{2} \rightarrow C_2 = -\frac{73}{24}$

$\boxed{y = e^{-2t}\left(-\frac{1}{4}\cos 12t - \frac{73}{24}\sin 12t\right) + \frac{1}{4}\cos 12t + 3\sin 12t}$