MAT2705-01/04 05S Quiz 6 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

$y'' + y' - 6y = 0$; $y_1 = \exp(2x)$, $y_2 = \exp(-3x)$; $y(0)=7$, $y'(0) = -1$. [x is the independent variable]

a) Verify that y_1 and y_2 are solutions of the differential equations by back-substitution (by hand).

b) Write down the general solution for y.

c) State the linear system of equations that result from imposing the initial conditions on y, and then write down the matrix form [**Ac** = **b**] of those equations, where **c** is the column vector of unknown coefficients.

d) Solve this system by row reduction (technology) or by using the inverse matrix (memory or technology) and state your final value for y.

e) OPTIONAL only if you are sure everything above is correct: find the exact value of x where this function assumes its minimum value (calc1).

a) $y_1 = e^{2x}$ $\quad y_1' = 2e^{2x}.$ $\quad y_1'' = 4e^{2x}$ $\quad y_1'' + y_1' - 6y_1 = (4e^{2x}) + (2e^{2x}) - 6(e^{2x}) = e^{2x}(4+2-6) = 0$ ✓

$y_2 = e^{-3x}$ $\quad y_2' = -3e^{-3x}$ $\quad y_2'' = 9e^{-3x}$ $\quad y_2'' + y_2' - 6y_2 = (9e^{-3x}) + (-3e^{-3x}) - 6(e^{-3x}) = e^{-3x}(9-3-6) = 0$ ✓

b) $\boxed{y = c_1 e^{2x} + c_2 e^{-3x}}$ $\longrightarrow$ $y(0) = \boxed{\begin{aligned} c_1 + c_2 &= 7 \\ 2c_1 - 3c_2 &= -1 \end{aligned}}$ $\rightarrow$ $\boxed{\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}}$ matrix form of eqns

c) $y' = 2c_1 e^{2x} - 3c_2 e^{-3x}$ $\rightarrow$ $y'(0) =$

eqns

d) $\begin{bmatrix} 1 & 1 & 7 \\ 2 & -3 & -1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{cc} c_1 & c_2 & \\ \end{array} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \end{bmatrix} \rightarrow \begin{aligned} c_1 &= 4 \\ c_2 &= 3 \end{aligned}$

your choice

or $\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}^{-1} = \dfrac{\begin{bmatrix} -3 & -1 \\ -2 & 1 \end{bmatrix}}{-3-2} = \dfrac{1}{5}\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$ $\quad \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \dfrac{1}{5}\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 7 \\ -1 \end{bmatrix} = \dfrac{1}{5}\begin{bmatrix} 20 \\ 15 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$\boxed{y = 4e^{2x} + 3e^{-3x}}$

e) OPTIONAL : $\quad y' = 8e^{2x} - 9e^{-3x} = 0$

$\dfrac{e^{3x}}{8}\left[8e^{2x} = 9e^{-3x}\right] \rightarrow e^{5x} = \dfrac{9}{8} \rightarrow \boxed{x = \frac{1}{5}\ln\left(\frac{9}{8}\right)}$ $\quad \approx 0.02356$

if we also want to know the yvalue there:

$y = 4e^{\frac{2}{5}(\ln 9/8)} + 3e^{-\frac{3}{5}(\ln 9/8)} = 4\left(\frac{9}{8}\right)^{2/5} + 3\left(\frac{9}{8}\right)^{-3/5} \approx 6.98827$

rules of exps/lns !!

graph it to make sure it is a local minimum:

global since only critical point on whole real line.