MAT2705-01/04 05S Quiz 3 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① Using one of the two shorthand notations: $R_1 \leftrightarrow R_2$ swaprow(1,2) $R_1 \to 2R_1$ mulrow(1,2) $R_1 \to -3R_2 + R_1$ addrow(2,1,-3)

reduce the matrix $\begin{bmatrix} 6 & 5 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ to its fully reduced row echelon form (rref) by a series of single row operations, recording each intermediate matrix and the row operation that created it.

② Consider a body that moves horizontally through a medium whose resistance is proportional to the square of the velocity v, so that $dv/dt = -kv^2$. Show that $v(t) = \dfrac{v_0}{1+v_0kt}$.

4 OPTIONAL PART only for those students who are sure everything they did above is correct and who have extra time to kill:
If we choose time units so that $v(1) = v_1$ is the value of the velocity at time $t=1$, solve for k and re-express $v(t)$ in terms of v_1, simplifying your result.

①
$$\begin{bmatrix} 6 & 5 & 4 \\ 1 & 2 & 3 \end{bmatrix} \xrightarrow[\text{swaprow}(1,2)]{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & 3 \\ \underset{-6}{6} & \underset{-12}{5} & \underset{-18}{4} \end{bmatrix} \xrightarrow[\text{addrow}(1,2,-6)]{R_2 \to -6R_1 + R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -14 \end{bmatrix} \xrightarrow[\text{mulrow}(2,-1/7)]{R_2 \to -\frac{1}{7}R_2} \begin{bmatrix} \underset{-0}{1} & \underset{-2}{2} & \underset{-4}{3} \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow[\text{addrow}(2,1,-2)]{R_1 \to -2R_2 + R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

②
$\dfrac{dv}{dt} = -kv^2$ separable

$v^{-2}dv = -k\,dt$ separate

$\int v^{-2}dv = -\int k\,dt$ } integrate

$-v^{-1} = -kt + C$ }

$v^{-1} = kt - C$ } solve for v

$v = \dfrac{1}{kt - C}$ }

$v_0 = v(0) = \dfrac{1}{0-C} = -\dfrac{1}{C}$ } impose I.C.

$\to C = -\dfrac{1}{v_0}$ }

$v = \dfrac{1}{kt - \left(-\frac{1}{v_0}\right)} = \dfrac{1}{kt + \frac{1}{v_0}} = \dfrac{v_0}{1 + ktv_0}$ ✓

(derivation approach)

or

$v = v_0(1 + kv_0t)^{-1} \longrightarrow \dfrac{dv}{dt} = -kv^2$

$\dfrac{dv}{dt} = v_0(-1)(1+kv_0t)^{-2}(kv_0)$

$= -\dfrac{kv_0^2}{(1+kv_0t)^2} \longrightarrow$

$\downarrow$

$-\dfrac{kv_0^2}{(1+kv_0t)^2} \overset{?}{=} -k\left(\dfrac{v_0}{1+kv_0t}\right)^2$

$= -\dfrac{kv_0^2}{(1+kv_0t)^2}$ ✓

(backsubstitute check approach)

$v(0) = \dfrac{v_0}{1+0} = v_0$ ✓

v_0 is understood to be $v(0)$ so this too must be checked in this alternative approach.

OPTION:

$v(1) = \dfrac{v_0}{1+kv_0} = v_1$

$\dfrac{1+kv_0}{v_0} = \dfrac{1}{v_1}$

$1 + kv_0 = \dfrac{v_0}{v_1}$

$kv_0 = \dfrac{v_0}{v_1} - 1 = \dfrac{v_0 - v_1}{v_1}$

$\boxed{k = \dfrac{v_0 - v_1}{v_0v_1}}$

$v = \dfrac{v_0}{1 + \left(\frac{v_0 - v_1}{v_0v_1}\right)v_0t}$

$= \dfrac{v_0}{1 + \frac{(v_0 - v_1)}{v_1}t}$

$= \boxed{\dfrac{v_1v_0}{v_1 + (v_0 - v_1)t} = v}$