MAT2705-01/04 05S Quiz 2 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

$$\frac{dy}{dx} = \frac{2xy+2x}{x^2+1}$$

a) Is this separable? Why or why not?
b) Is this linear in y (as the unknown)? Why or why not?
c) Is this linear in x (as the unknown)? Why or why not?
d) This DEQ can be solved in two different ways. Pick one method to solve it, reporting your final answer in the form y = "y(x)".
e) Repeat part d) with a second method.
f) Do your two solutions agree?
g) Now check your solution (one of them) by backsubstitution into the DEQ.
h) Find the solution for which y(0) = 0, reporting your final answer in the form y = "y(x)".
i) What movie would you like to see this weekend?

a) $\frac{dy}{dx} = \frac{2xy+2x}{x^2+1} = \left(\frac{2x}{x^2+1}\right)(y+1)$ $\boxed{\text{separable}}$

b) $\frac{dy}{dx} = \left(\frac{2x}{x^2+1}\right) y + \left(\frac{2x}{x^2+1}\right)$ rhs is linear in y, so $\boxed{\text{linear in } y}$

c) $\frac{dx}{dy} = \left(\frac{x^2+1}{x}\right)(y+1)$ rhs is not linear in x so $\boxed{\text{not linear in } x}$

d) separable
e) method: $\frac{dy}{dx} = (y+1)\left(\frac{2x}{x^2+1}\right)$ (or use technology)

$\frac{dy}{y+1} = \frac{2x\,dx}{x^2+1} \rightarrow \int \frac{dy}{y+1} = \int \frac{\overbrace{2x\,dx}^{=du}}{\underbrace{x^2+1}_{=u}} = \int \frac{du}{u} = \ln|u| + C = \ln(x^2+1) + C$

$\ln|y+1| = \ln(x^2+1) + C_1$

$e^{\ln|y+1|} = e^{\ln|x^2+1| + C_1} = e^{C_1} e^{\ln(x^2+1)}$

$|y+1| = e^{C_1}(x^2+1)$

$y+1 = \pm e^{C_1}(x^2+1) = C(x^2+1)$

$\boxed{y = -1 + C(x^2+1)}$ gen. soln.

NOTE: $e^{A+B} = e^A e^B \neq e^A + e^B$

$e^{AB} = (e^A)^B$

g) $y = -1 + C(x^2+1) \rightarrow y+1 = C(x^2+1)$

$\frac{dy}{dx} = 0 + C(2x+0) = 2Cx$

$\frac{dy}{dx} = \frac{2x}{x^2+1}(y+1)$ substitute for y everywhere!

$2Cx = \frac{2x}{\cancel{(x^2+1)}} C\cancel{(x^2+1)}$ simplify LHS & RHS independently

$= 2Cx$ ✓

i) ☺

e) linear in y
d) method: $\left[\frac{dy}{dx} + \left(-\frac{2x}{(x^2+1)}\right) y = \frac{2x}{x^2+1}\right] \longrightarrow \frac{d}{dx}\left(y(x^2+1)^{-1}\right) = \frac{2x}{(x^2+1)^2}$

standard linear form first

$\int -\frac{2x}{x^2+1}dx = -\ln(x^2+1)$ (see above)

$e^{-\ln(x^2+1)} = \left(e^{\ln(x^2+1)}\right)^{(-1)} = (x^2+1)^{-1}$

$y(x^2+1)^{-1} = \int \frac{\overbrace{2x\,dx}^{=du}}{\underbrace{(x^2+1)^2}_{=u^2}} = \int u^{-2}du = -u^{-1} + C_2 = -\frac{1}{x^2+1} + C_2$

$\boxed{y = -1 + C_2(x^2+1)}$ gen. soln.

f) They agree with $C = C_2$.

h) $y(0)=0 \rightarrow x=0, y=0 \rightarrow y = -1 + C(x^2+1)$

$0 = -1 + C(0+1) = -1 + C$

$C = 1$

$\boxed{y = -1 + (x^2+1) = x^2}$