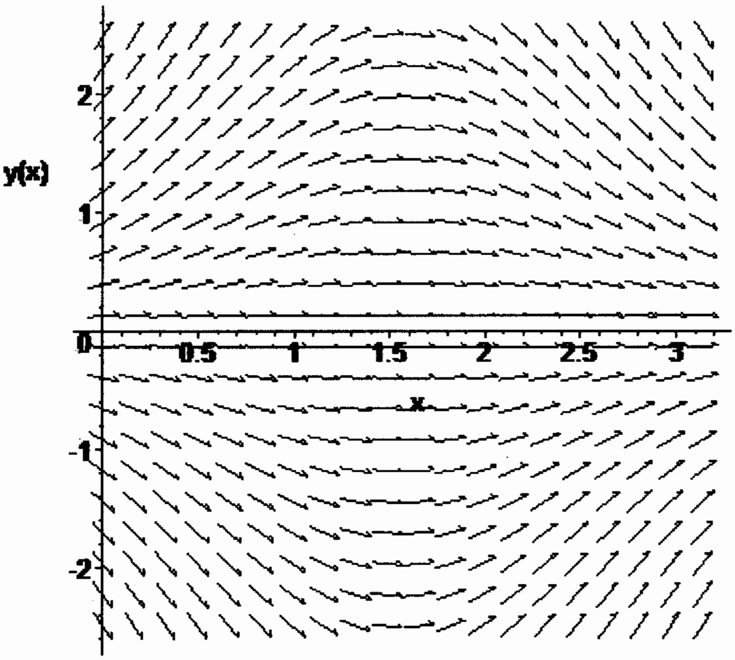
MAT2705-01/04 05S Quiz 1   Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

$$\frac{dy}{dx} = y\cos x\,, \quad y(\pi/2) = -2$$



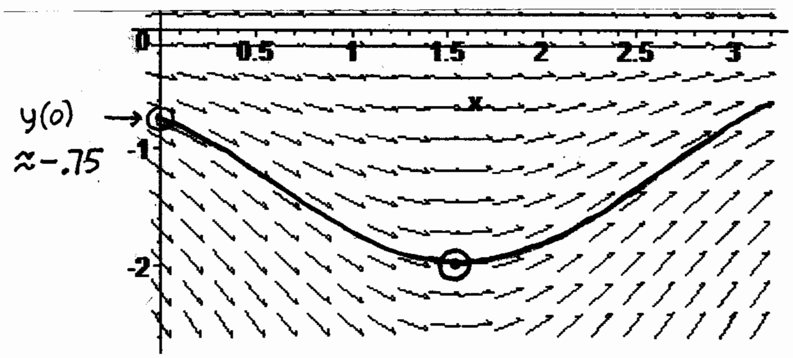
a) Locate the initial data point in the direction field with "⊙" and draw in the solution curve passing through it. Estimate roughly the value of $y(0)$ for this curve.

b) Find the general solution $y = y(x)$ of this D.E., showing all steps in the process.

c) Impose the initial condition on this solution to solve the IVP.

d) Give both an exact (no decimals) and numerical value for $y(0)$ for this solution. How does it compare to your estimate from part a)?

e) Check your solution to c) by back substitution.



a) $\pi/2 \approx 1.57 \to$ point : $(1.57, -2)$

b) separable: $\dfrac{dy}{dx} = y\cos x$

$$\int \frac{dy}{y} = \int \cos x\, dx$$

$$\ln|y| = \sin x + C_1$$

$$|y| = e^{\ln|y|} = e^{\sin x + C_1} = e^{C_1} e^{\sin x}$$

$$y = \underbrace{\pm e^{C_1}}_{C} e^{\sin x} = C e^{\sin x}$$

Gen. soln: $\boxed{y = C e^{\sin x}}$

c) $-2 = y(\tfrac{\pi}{2}) = C e^{\sin\frac{\pi}{2}} = C e^{1} = Ce$

$\to C = -2e^{-1} \to \boxed{y = -2e^{-1} e^{\sin x} = -2e^{\sin x - 1}}$

d) $y(0) = -2e^{\sin 0 - 1} = -2e^{0-1} = \boxed{-2e^{-1} \approx -0.7358}$ my estimate was not bad!

e) $y = -2e^{\sin x - 1}$ $\dfrac{dy}{dx} = \dfrac{d}{dx}(-2e^{\sin x - 1}) = -2\dfrac{d}{dx}e^{\sin x - 1} = -2e^{\sin x - 1}\underbrace{\dfrac{d}{dx}(\sin x - 1)}_{\cos x}$

$\dfrac{dy}{dx} = y\cos x \to -2e^{\sin x - 1}\cos x = (-2e^{\sin x - 1})\cos x$ ✓

not requested: [final check: $y(\tfrac{\pi}{2}) = -2e^{\sin\frac{\pi}{2} - 1} = -2e^{1-1} = -2e^{0} = -2$ ✓]