MAT2705-01/04 05S Final Exam   Print Name (Last, First) ____________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers exact (no decimal approximations, if possible). [See long instructions on reverse].

① $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 26x = 82\cos 4t - 82\sin 4t$, $x(0) = 5$, $x'(0) = 0$

a) Solve this initial value problem, showing all steps.

b) Express the driving function in this deq in phase-shifted cosine form. It would be a good idea to first make a coefficient-plane plot that illustrates your calculation. Then use technology to plot the difference of the original function and your rewritten form which should be zero. Is it?

c) Plot your solution from part a) together with the steady state solution in an appropriate window using technology that shows them merging together as the transient dies away. Sketch what you see labeling the axes with variable names and numbered tickmarks and labeling the two curves.

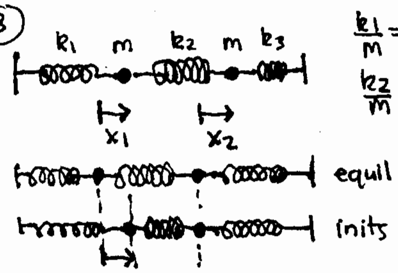
② $\underline{A} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

a) Find the eigenvalues of $\underline{A}$, using technology to evaluate the required determinant and to solve the condition.

b) For the smallest eigenvalue, find a basis $\{\underline{E}_1, \ldots\}$ of the associated eigenspace by hand using technology only for the required row reduction.

c) Evaluate the matrix product of $\underline{A}$ times the vector $[-1,2,1]$ and confirm from this result that this vector is an eigenvector of the matrix.

d) Can this vector $[-1,2,1]$ be expressed as a linear combination of the eigenvectors you found in part b)? Set up and solve a linear system of equations to answer this question and if the answer is yes, express the given vector as an explicit linear combination of those eigenvectors.

③


$\frac{k_1}{m} = \frac{k_3}{m} = 4$

$\frac{k_2}{m} = 6, m = 1$

$x_1'' = -10x_1 + 6x_2$ $x_1(0) = 1$, $x_1'(0) = 0$

$x_2'' = 6x_1 - 10x_2$ $x_2(0) = 0$, $x_2'(0) = 0$

The second mass is held at equilibrium while the first mass is pulled to the right 1 unit. Then both are released. Can you guess which directions the two masses will begin moving? [OPTIONAL] Explain.

a) Write both the deqs and the inits in matrix form ($\underline{x}'' = \underline{A}\,\underline{x}$ etc) identifying the coefficient matrix.

b) Find the eigenvalues of $\underline{A}$ by hand, ordering them so that $|\lambda_1| < |\lambda_2|$.

c) Find the corresponding eigenvectors $\underline{E}_1$ and $\underline{E}_2$.

d) Write down the new differential equations for the new variables $\underline{y}$ related to $\underline{x}$ by $\underline{x} = \underline{B}\,\underline{y}$ where $\underline{B} = \text{augment}(\underline{E}_1, \underline{E}_2)$. Then solve them, showing all work.

e) Express the general solution in terms of x_1 and x_2.

f) Solve the initial value problem and state your results for x_1 and x_2, as well as in vector form as a linear combination of the eigenvectors to identify the two modes of the system.

g) The collective motion of the system is periodic. What is the (smallest) period of the system motion? Plot x_1 and x_2 versus t for one such period (after first checking that it actually repeats over a window of 2 such periods) using technology and make a rough sketch of what you see, labeling the axes with numbered tickmarks and identifying the two functions by their variable names and the horizontal axis by its variable name.

OPTIONAL. If you considered the optional question above, does your plot agree with your intuition? Explain.