MAT1505-05 04S Quiz 7 Not! Print Name (Last, First) ______________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① $S=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n\cdot 2^n}$

a) Why does this converge by the alternating series test?

b) Apply the ratiotest. Does this converge absolutely?

c) Using the alternating series remainder estimate, what is the minimum number N of terms needed so that the partial sum S_N is accurate to 4 decimal places (i.e. the remainder is less than $\frac{1}{2}\times 10^{-4}$ in absolute value)?
[Hint: look at the sequence of numerical terms with Maple.]

d) If you round off S_N to 4 decimal places, does this agree with the 4 decimal place round off of Maple's 10 digit result for the infinite sum S ?

e) Is S_N or $\text{round}_4(S_N)$ closer to S ? (calculate the two differences)
If S is used in a step in a larger calculation, which number makes sense to use as an approximation for S: S_N or its 4 decimal place roundoff?

① a) $|a_n| = \frac{1}{n\cdot 2^n}$ is a <u>decreasing</u> sequence with <u>0</u> as its limit, the two conditions of the alternating series test that guarantee convergence.

b) $$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{1}{(n+1)\,2^{n+1}}}{\frac{1}{n2^n}} = \left(\frac{n}{n+1}\right)\left(\frac{2^n}{2^{n+1}}\right) = \left(\frac{1}{1+\frac{1}{n}}\right)\left(\frac{1}{2}\right)$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left(\frac{1}{1+\underbrace{\tfrac{1}{n}}_{\to 0}}\right)\left(\frac{1}{2}\right) = \underline{\frac{1}{2} < 1}$$ so this converges absolutely

c) See reverse side of sheet. With technology:

$a_{11} \approx 0.0000\ 4438\ 9204\ 55$ $\qquad S_{10} \approx 0.4054\ 3464\ 78$
$< 0.0000\ 5$

Since $|a_{11}|$ is larger than $|S-S_{10}|$, the absolute value of the remainder, our truncation error for N=10 is smaller than $\frac{1}{2}\times 10^{-4}$.

d) $S_{10} \to 0.4054$ but $S = \ln\left(\frac{3}{2}\right) \approx 0.4054\ 6510\ 81 \to 0.4055$
so our rounded off S_{10} has the wrong 4th digit

e) $S - .4054 \approx 0.0000\ 65$
$S - .4054\ 3464 \approx 0.0000\ 30$ ← unrounded result closer to true value.