MAT1505-05 04S Quiz 5     Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

Do the upper or lower version of this problem but not both:

① a) Evaluate $\int \frac{y}{\sqrt{y-2}}\,dy$ or $\int x e^{-x^2}dx$ using a u-substitution

b) Use part a) to evaluate the improper integral: $\int_2^6 \frac{y}{\sqrt{y-2}}\,dy$ or $\int_0^\infty x e^{-x^2}dx$ starting from a correctly stated limit expression.

② a) Write down an integral for the arclength of the curve $y = \ln x$ between 1 and 2 and simplify the integrand as much as possible without attempting to integrate it.

b) Approximate the integral with the $n=2$ division Simpson rule (recall that for each pair of subintervals from $x_0$ to $x_1$ to $x_2$, the weighting coefficients are $\frac{1}{3}y_0 + \frac{4}{3}y_1 + \frac{1}{3}y_2$).

c) Evaluate the integral exactly with technology (write down the result) and then give its decimal approximation. They should agree to 3 significant digits. Do they? (simpson & exact)

① a) $\int \frac{y}{(y-2)^{1/2}}\,dy = \int \frac{(u+2)\,du}{u^{1/2}} = \int u^{1/2} + 2u^{-1/2}\,du$

$u = y-2 \to y = u+2$

$\frac{du}{dy} = 1$

$du = dy$

$= \frac{2}{3}u^{3/2} + 2(2u^{1/2}) + C$

$= \boxed{\frac{2}{3}(y-2)^{3/2} + 4(y-2)^{1/2} + C}$

$\int x e^{-x^2}dx = \int e^u\left(-\frac{du}{2}\right) = -\frac{1}{2}e^u + C$

$u = -x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \boxed{-\frac{1}{2}e^{-x^2} + C}$

b) $\int_2^6 \frac{y}{(y-2)^{1/2}}dy = \lim_{t\to 2^+}\int_t^6 \frac{y}{(y-2)^{1/2}}dy =$

$= \lim_{t\to 2^+}\left(\frac{2}{3}(y-2)^{3/2} + 4(y-2)^{1/2}\Big|_t^6\right)$

$= \lim_{t\to 2^+}\left(\frac{2}{3}4^{3/2} + 4\,4^{1/2} - \frac{2}{3}(t-2)^{3/2} - 4(t-2)^{1/2}\right) = \frac{2}{3}\cdot 8 + 8 - 0$

$= 8\left(\frac{5}{3}\right) = \boxed{\frac{40}{3}}$

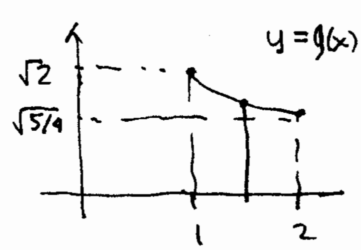
$\int_0^\infty x e^{-x^2}dx = \lim_{t\to\infty}\int_0^t x e^{-x^2}dx = \lim_{t\to\infty}\left(-\frac{1}{2}e^{-x^2}\Big|_0^t\right)$

$= \lim_{t\to\infty}\left(-\frac{1}{2}e^{-t^2} + \frac{1}{2}\right) = 0 + \frac{1}{2} = \boxed{\frac{1}{2}}$

② a) $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$, $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1}{x^2} = \frac{x^2+1}{x^2}$,

$L = \int_1^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^2 \sqrt{\frac{x^2+1}{x^2}}\,dx = \boxed{\int_1^2 \underbrace{\frac{\sqrt{x^2+1}}{x}}_{g(x)}\,dx}$




$\Delta x = \frac{2-1}{2} = 1/2$

b) $S_2 = \frac{1}{2}\cdot\frac{1}{3}\left(g(1) + 4g(1.5) + g(2)\right)$

$\approx \boxed{1.223}$

c) > int(sqrt(x^2+1)/x, x=1..2);

$-2 + \operatorname{arctanh}\left(\frac{\sqrt{2}}{2}\right) + \sqrt{5} - \operatorname{arctanh}\left(\frac{\sqrt{5}}{5}\right)$

> evalf(%);     $\boxed{1.222016177}$

→ yes, they agree to 3 significant digits.

[hyperbolic functions pop up in applications you should at least be aware of them. see Stewart section 3.9, in particular eqn 5 on p253.]