MAT1505-05 04S Quiz 4 Print Name (Last, First) ____________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① a) Evaluate $\int x e^{-x/2}\,dx$ by hand.
b) Use the result of part a) to evaluate $I = \int_0^4 x e^{-x/2}\,dx$ exactly (no decimal pts).
c) Make a rough sketch of the region whose area is represented by this integral (technology).
d) Evaluate the trapezoidal approximation T_4 of this integral ($n=4$ divisions)
e) Evaluate I numerically (decimal equivalent) and the error $E_4 = I - T_4$.
f) Compute the error bound $|E_n| \le \dfrac{K(b-a)^3}{12n^2}$, where $n=4$ and K is the maximum absolute value of the second derivative of the integrand on the interval $[a,b]$ of integration (you may use technology to do this). Is E_4 within this error bound?
g) What value of n is required to make the error less than $\frac{1}{2}\times 10^{-4}$?

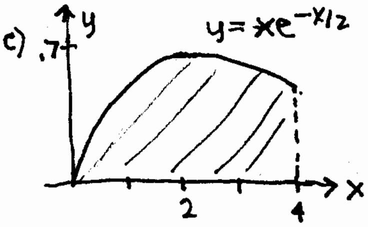
① a) $\int \underbrace{x}_{u}\,\underbrace{e^{-x/2}\,dx}_{dv} = \underbrace{x}_{u}(\underbrace{-2e^{-x/2}}_{v}) - \int (\underbrace{-2e^{-x/2}}_{v})\,\underbrace{dx}_{du} = -2xe^{-x/2} + 2\underbrace{\int e^{-x/2}\,dx}_{\frac{e^{-x/2}}{-1/2}}$

$u = x \quad dv = e^{-x/2}dx$
$du = dx \quad v = \int e^{-x/2}dx = \dfrac{e^{-x/2}}{-1/2}$

$= \boxed{-2xe^{-x/2} - 4e^{-x/2} + C}$
$= \boxed{-(2x+4)e^{-x/2} + C}$

b) $\int_0^4 xe^{-x/2}dx = -(2x+4)e^{-x/2}\Big|_0^4 = -12e^{-2} + 4 = \boxed{4 - 12e^{-2}}$

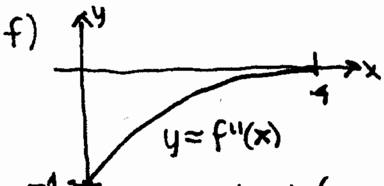
c)



d) $n=4$, $\Delta x = \frac{4-0}{4} = 1$, $T_4 = \Delta x\left[\frac{1}{2}f(0) + f(1) + f(2) + f(3) + \frac{1}{2}f(4)\right]$
$f(x) = xe^{-x/2} \longrightarrow$ $= 1\left[\frac{1}{2}(0) + e^{-1/2} + 2e^{-1} + 3e^{-3/2} + 2e^{-2}\right]$
$\approx \boxed{2.2824}$

e) $\boxed{I \approx 2.3760} \to \boxed{E_4 \approx 0.0936}$

f)



$f(x) = xe^{-x/2}$
$f'(x) = 1e^{-x/2} + xe^{-x/2}(-\frac{1}{2}) = (1-\frac{x}{2})e^{-x/2}$
$f''(x) = (-\frac{1}{2})e^{-x/2} + (1-\frac{x}{2})e^{-x/2}(-\frac{1}{2}) = (-\frac{1}{2}-\frac{1}{2}+\frac{x}{4})e^{-x/2} = (\frac{x}{4}-1)e^{-x/2}$

clearly $(\max|f''(x)|) = |f''(0)| = |-1| = \boxed{1 = K}$

$|E_4| \le \dfrac{1\cdot(4)^3}{12\cdot 4^2} = \dfrac{4}{12} = \dfrac{1}{3} \approx .33$ $\boxed{\text{yes}}$, E_4 is less than 1/3 this error bound.

g) $|E_n| \le \dfrac{4^3}{12n^2} = \dfrac{1}{3}\dfrac{4^2}{n^2} < \dfrac{1}{2}10^{-4} \to \dfrac{2}{3}4^2 10^4 < n^2 \to \underbrace{\sqrt{\tfrac{2}{3}}\cdot 4\cdot 10^2}_{326.6} < n \to \boxed{n = 327}$ (smallest value)

[compare with Simpson's rule: $|E_n| \le \dfrac{K_S(b-a)^5}{180n^4} = \dfrac{2.84}{n^4} < \frac{1}{2}10^{-4} \to n > 15.4 \to \underline{\underline{16}}$ next even integer
$K_S = \max|f^{(4)}(x)| = |f^{(4)}(0)| = |-\frac{1}{2}| = \frac{1}{2}$ and $E_{16} = 10^{-5}$]