MAT1505-05 04S Quiz 3 Print Name (Last, First) _____ 10

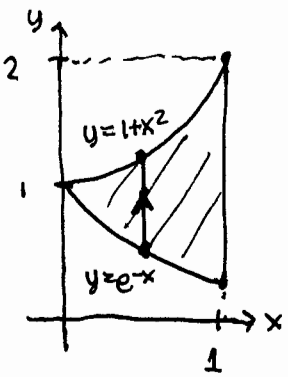
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① a) $y=x^2+1$, $y=e^{-x}$, $x=0$, $x=1$. Sketch the region enclosed by these curves, showing a typical cross-section of the shaded region and labeling all pertinant information in the diagram. Then find the exact area of the region.

b) Evaluate the definite integral you wrote down for part a) either with Maple or your graphing calculator. What is your result? Does it agree with the numerical value of the exact number you found for part a)?

c) Suppose you revolve this region about the x-axis. Make a new sketch illustrating this situation. Setup but do not evaluate an integral representing its volume.

① a)



$$\text{Area} = \int_0^1 (1+x^2) - e^{-x}\,dx$$

$$= \underbrace{\int_0^1 1+x^2\,dx}_{x+\frac{x^3}{3}\Big|_0^1} - \underbrace{\int_0^1 e^{-x}\,dx}_{\substack{u=-x \\ \frac{du}{dx}=-1 \\ du=-dx}} = \int_{x=0}^{x=1} e^u(-du) = -e^u\Big|_{x=0}^{x=1}$$

$$= 1+\tfrac{1}{3}-0 \qquad = -e^{-x}\Big|_0^1 = -e^{-1}+e^0$$

$$= \tfrac{4}{3} \qquad = 1-e^{-1}$$

$$= \frac{4}{3}-(1-e^{-1}) = \boxed{\tfrac{1}{3}+e^{-1}} \approx 0.7012$$

b)
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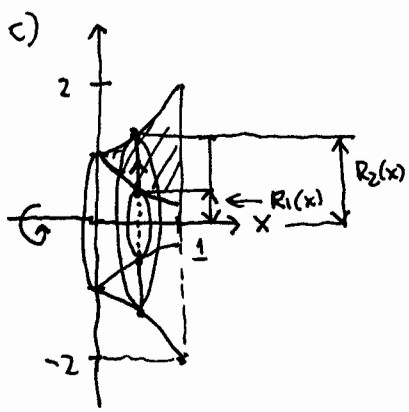
> int(1+x^2-exp(-x), x=0..1);
        e^(-1) + 1/3
> evalf(%);
```
0.7012127745 yes, they agree

c)



$$A(x) = \pi(R_2(x)^2 - R_1(x)^2)$$

$$= \pi((1+x^2)^2 - (e^{-x})^2)$$

$$= \pi((1+x^2)^2 - e^{-2x})$$

$$\text{Volume} = \int_0^1 A(x)\,dx$$

$$= \int_0^1 \pi((1+x)^2 - e^{-2x})\,dx$$

$$= \pi\int_0^1 (1+2x+x^2-e^{-2x})\,dx$$