

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① A honeybee population starts with 100 bees and increases at a rate of  $n'(t)$  bees per week. What does  $100 + \int_0^{15} n'(t) dt$  represent? Evaluate this expression symbolically before answering the question in a clear and complete English sentence.

② a) Evaluate this integral using the substitution rule:

$$\int_0^a x \sqrt{x^2+a^2} dx, \quad a > 0 \text{ constant.}$$

b) Can you check your answer with Maple? If so, do they agree? If you cannot check with Maple, does the approximate numerical value of this integral when  $a=1$  equal 0.609?

①  $100 + \int_0^{15} n'(t) dt = n(15)$  This is the total population of bees at the end of 15 weeks.  
 (starting population) (increase in population during first 15 weeks)

② a)  $\int_0^a x \sqrt{x^2+a^2} dx = \int_{x=0}^{x=a} u^{1/2} \left(\frac{du}{2}\right) = \frac{1}{2} \int_{x=0}^{x=a} u^{1/2} du = \frac{1}{2} \left(\frac{u^{3/2}}{3/2}\right) \Big|_{x=0}^{x=a} = \frac{1}{3} (x^2+a^2)^{3/2} \Big|_0^a$

$u = x^2+a^2$   
 $\frac{du}{dx} = 2x$   
 $du = 2x dx$   
 $\frac{du}{2} = x dx$

or  $\int_{a^2}^{2a^2} u^{1/2} du = \frac{1}{2} \frac{u^{3/2}}{3/2} \Big|_{a^2}^{2a^2} = \frac{1}{3} (2^{3/2} a^3) - \frac{1}{3} a^3 = \frac{1}{3} (2^{3/2}-1) a^3$

$x=a \rightarrow u=a^2+a^2=2a^2$   
 $x=0 \rightarrow u=a^2$

b) `> int(x * sqrt(x^2+a^2), x=0..a);` same result except for complex sign  $\text{sgn}(a) = 1$

`> evalf((2^(3/2)-1)/3);` } or use g-calculator  
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