MAT1505-04 04S Quiz 1 Print Name (Last, First) ____________ /10

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① Write as a single integral in the form $\int_a^b f(x)\,dx$: $\int_{-2}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx$.
Support your work with a diagram and explanation.

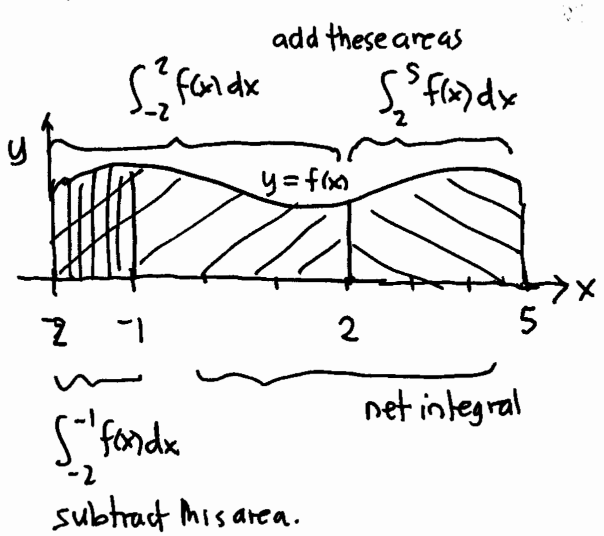
② $\int_0^2 x^2\,dx$ a) Make a completely labeled diagram illustrating the $n=2$ middlebox approximation (i.e. midpoint rectangle) to this integral.

b) Evaluate the middlesum M_2 corresponding to this approximation, showing the factors in each product before summing to a final decimal result.

c) Now consider the right endpoint rectangular approximation for any n. Evaluate the right endpoint x_i of the ith subinterval and then the rightsum R_n of the area of these n rectangles, using Σ notation, showing that you get the result $\frac{8}{n^3}\left(\sum_{i=1}^{n} i^2\right)$.

d) Use the formula $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ to evaluate this sum and then take its limit to find the value of the integral. How does it compare with M_2?

①



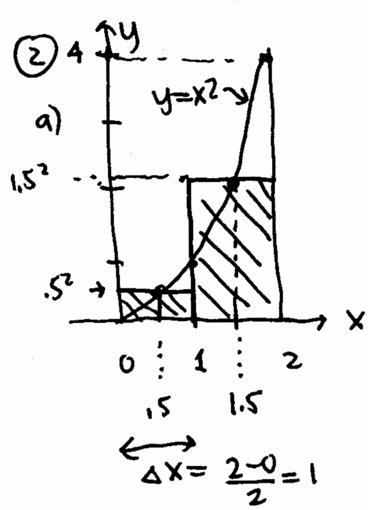
$$\underbrace{\int_{-2}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx} - \int_{-2}^{-1} f(x)\,dx$$

$$= \underbrace{\int_{-2}^{5} f(x)\,dx} - \int_{-2}^{-1} f(x)\,dx$$

$$= \cancel{\int_{-2}^{-1} f(x)\,dx}^{\,0} + \int_{-1}^{5} f(x)\,dx - \cancel{\int_{-2}^{-1} f(x)\,dx}^{\,0}$$

$$= \boxed{\int_{-1}^{5} f(x)\,dx}$$

② a)



b) $M_2 = 1\cdot .5^2 + 1\cdot 1.5^2 = .25 + 2.25 = \boxed{2.50}$

c) $\Delta x = \frac{2-0}{n} = \frac{2}{n}$ $\quad x_i = 0 + i\Delta x = i\left(\frac{2}{n}\right) = \frac{2i}{n}$

$R_n = \sum_{i=1}^{n} x_i^2 \Delta x = \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right) = \sum_{i=1}^{n} \frac{8i^2}{n^3} = \boxed{\frac{8}{n^3}\left(\sum_{i=1}^{n} i^2\right)}$ ✓

$\lim_{n\to\infty} R_n = \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) = \frac{8\,(n+1)(2n+1)}{6\,n^2} = \boxed{\frac{4}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}$

$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{4}{3}\left(1+\cancel{\tfrac{1}{n}}_{\to 0}\right)\left(2+\cancel{\tfrac{1}{n}}_{\to 0}\right) = \frac{4\cdot 2}{3} = \boxed{\frac{8}{3}} = 2\frac{2}{3}$
≈ 2.667

2.5 is not bad for such a rough approximation.