MAT1505-05 04S Test 2   Print Name (Last, First) ______________________________|___

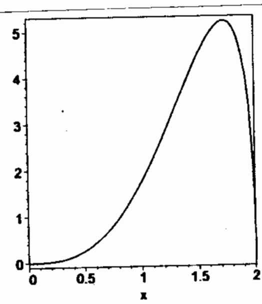
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers exact (no decimal approximations, if possible). [See long instructions on reverse].

① $\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx$

a) Why is this an improper integral?

b) Using limit notation, evaluate this integral.

c) What happens to this problem if the square root is replaced by the 3/2 power? Explain.

② 

$f(x) = x^3\sqrt{4-x^2}$

a) Eyeballing its graph, make a very rough estimate of the integral of f from 0 to 2, and from it calculate the average value f_{avg} of f on that interval.

b) Using an obvious variable substitution $u = 4-x^2$, completely re-express $\int_0^2 f(x)\,dx$ as a new definite integral in terms of the variable u (in simplest form).

c) Now evaluate this new integral and from it calculate the exact average value of f on this interval. Does it agree with your guesstimate?

③

time t (min)	0	1	2	3	4	5	6
speed $v(t)$ (mi/hr)	40	42	45	49	52	54	56

Simpson's Rule (n even):

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n)\right]$$

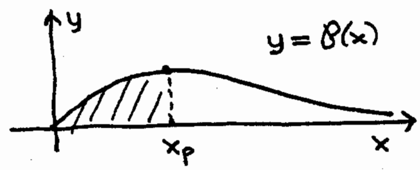
The speedometer reading on a car was observed at 1 minute intervals and recorded in the table. Use Simpson's rule to estimate the distance traveled by the car. Work the problem entirely in units of hours and miles or you will not get a reasonable answer. [How far would the car go in 6 minutes at a constant speed of 50 mi/hr?] Is your answer reasonable?

④ $\rho(x) = k^2 x e^{-kx}$, $k > 0$.

a) Evaluate $\int k^2 x e^{-kx}\,dx$ (by hand, showing all work).

b) Show that $\rho(x)$ defines a probability distribution function for $0 \le x < \infty$ by showing that $\int_0^\infty \rho(x)\,dx = 1$ (use limit notation).

c) Find the value x_p at which $\rho(x)$ has its peak value.



d) Evaluate the probability that x assumes a value less than or equal to x_p:

$P(0 \le x \le x_p) = \int_0^{x_p} \rho(x)\,dx$. (use part a))

Give the exact value and its decimal approximation to 3 decimal places.