MAT1505 04S Test 1 Answers

① a) $\int (u^{1/4}+1)^2\,du = \int (u^{1/4})^2+2u^{1/4}+1\,du = \int (u^{1/2}+2u^{1/4}+1)\,du = \frac{u^{3/2}}{3/2}+2\frac{u^{5/4}}{5/4}+u+C$

$= \boxed{\frac{2}{3}u^{3/2}+\frac{8}{5}u^{5/4}+u+C}$

b) $\int_3^4 \frac{x\,dx}{x^2-4} = \int_{x=3\,(u=5)}^{x=4\,(u=12)} \frac{1}{u}\frac{du}{2} = \frac{1}{2}\ln|u|\Big|_{x=3\,(u=5)}^{x=4\,(u=12)} = \begin{cases} \frac{1}{2}\ln|u|\Big|_5^{12} \\ \frac{1}{2}\ln|x^2-4|\Big|_3^4 \end{cases}$

$u=x^2-4$ ($\neq 0$ in interval)

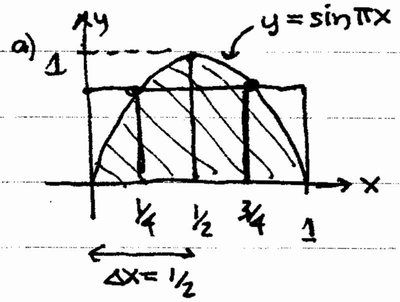
$\frac{du}{dx}=2x$

$du=2x\,dx$

$\frac{du}{2}=x\,dx$

$= \frac{1}{2}\ln 12-\frac{1}{2}\ln 5 = \boxed{\frac{1}{2}(\ln 12-\ln 5) = \frac{1}{2}\ln\frac{12}{5}} \approx 0.438$

② a)



b) $M_2 = [f(\tfrac{1}{4})+f(\tfrac{3}{4})]\Delta x = \frac{1}{2}\left(\sin\frac{\pi}{4}+\sin\frac{3\pi}{4}\right) = \frac{1}{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{\sqrt{2}} \approx \boxed{0.707}$

c) $A = \int_0^1 \sin\pi x\,dx = \int_{x=0\,(u=0)}^{x=1\,(u=\pi)} \sin u\frac{du}{\pi} = \frac{1}{\pi}(-\cos u)\Big|_{x=0\,(u=0)}^{x=1\,(u=\pi)}$

$u=\pi x$

$\frac{du}{dx}=\pi$

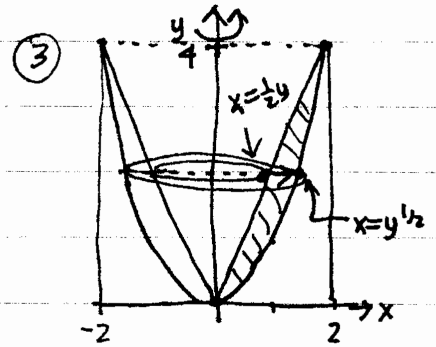
$du=\pi\,dx$

$= \begin{cases} -\frac{1}{\pi}\cos u\Big|_0^{\pi} = -\frac{1}{\pi}\cos\pi+\frac{1}{\pi}\cos 0 \\ -\frac{1}{\pi}\cos\pi x\Big|_0^1 = \frac{1}{\pi}(1+1) = \boxed{\frac{2}{\pi}} \end{cases}$

d) $A \approx 0.637$

Error $= \left(\frac{.707-.637}{.637}\right)100 \approx \boxed{11\%}$

③



$y=2x$, $y=x^2$] $x^2=2x$ $x^2-2x=0$ $x(x-2)=0$ $x=0,2$ $y=0,4$

$x=\frac{1}{2}y$

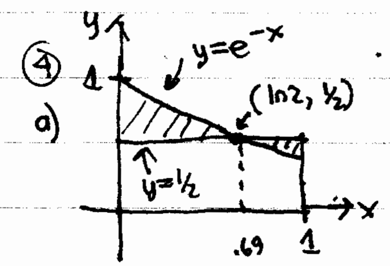
$x=y^{1/2}$

$V = \int_0^4 \pi[(y^{1/2})^2-(\tfrac{y}{2})^2]\,dy$

$= \pi\int_0^4 \left(y-\frac{y^2}{4}\right)dy = \pi\left(\frac{y^2}{2}-\frac{y^3}{12}\right)\Big|_0^4$

$= \pi\left(\frac{16}{2}-\frac{16}{3}\right) = \pi\cdot\frac{16}{6} = \boxed{\frac{8\pi}{3}} \approx 8.378$

④ a)



$e^{-x}=\frac{1}{2} \rightarrow \ln(e^{-x}) = \ln\frac{1}{2} \rightarrow -x=\ln\frac{1}{2} \rightarrow x=-\ln\frac{1}{2}=-\ln 2^{-1}=\boxed{\ln 2}$ ≈ 0.693

b) $\int_0^1 |e^{-x}-\tfrac{1}{2}|\,dx = \boxed{\int_0^{\ln 2}(e^{-x}-\tfrac{1}{2})\,dx+\int_{\ln 2}^1(\tfrac{1}{2}-e^{-x})\,dx}$

c)

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c)> Int( exp(-x)-1/2, x=0..ln(2)) - Int( exp(-x)-1/2, x=ln(2)..1);
 > value(%);
 > evalf(%);
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$\boxed{-\ln 2+\frac{1}{2}+e^{(-1)}} \approx 0.175$