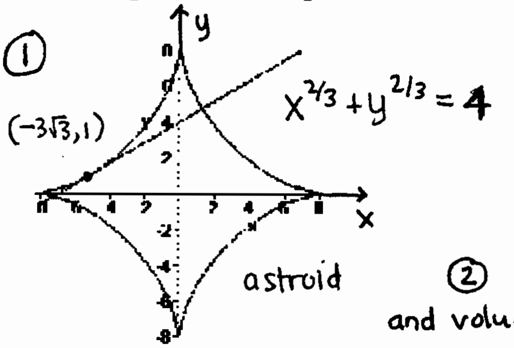
MAT1500-04/06 04F Quiz 6    Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

1




a) Show that the point $(-3\sqrt{3},1)$ lies on this curve.

b) Use implicit differentiation to evaluate the slope of the tangent line to the curve at this point. (exactly!)

c) Write an equation for the tangent line there, solving for y. (Does your slope value agree with an estimate from the diagram?)

2 For an ideal gas at constant temperature, the pressure P and volume V satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant the pressure is 500 kPa and the volume is 1000 cm³ and the volume is decreasing at a rate of 20 cm³/min. At what rate is the pressure increasing at this instant?

1 a) $3\sqrt{3} = 3\cdot 3^{1/2} = 3^{3/2}$ (always convert to power notation)

$\underbrace{(-3^{3/2})^{\frac{2}{3}}}_{(-1)^2 (3^{3/2})^{2/3}} + \underbrace{1^{2/3}}_{1} \overset{?}{=} 4$

$\underbrace{\underbrace{(-1)^2}_{1}\underbrace{(3^{3/2})^{2/3}}_{3^1} \quad 1}_{3+1=4 \checkmark}$

b) $\frac{d}{dx}\left[x^{2/3}+y^{2/3}=4\right]$

$\frac{d}{dx}x^{2/3}+\frac{d}{dx}y^{2/3} = \frac{d}{dx}4$

$\cancel{\tfrac{2}{3}}x^{-1/3}+\cancel{\tfrac{2}{3}}y^{-1/3}\frac{dy}{dx} = 0$

$y^{-1/3}\frac{dy}{dx} = -x^{-1/3}$

$\frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}}$

$\left.\frac{dy}{dx}\right|_{\substack{x=-3^{3/2}\\ y=1}} = \frac{-1^{1/3}}{(-3^{3/2})^{1/3}} = \frac{-1}{-3^{1/2}} = \boxed{\frac{1}{3^{1/2}}}$

$\approx 0.577$

c) $y-1 = \frac{1}{3^{1/2}}(x-(-3^{3/2})) \rightarrow \boxed{y = 1+\frac{1}{3^{1/2}}(x+3^{3/2})}$

$= 1+\frac{x}{3^{1/2}}+3$ or $\boxed{y = 4+\frac{x}{3^{1/2}}}$

slope estimate: from left to right: up about 8.5 units, over about 15 units: $\frac{8.5}{15}\approx 0.567$ pretty close!

2 $\left.\frac{dV}{dt}\right|_{\substack{P=500\\ V=1000}} = -20\,\frac{\text{cm}^3}{\text{min}} \rightarrow \left.\frac{dP}{dt}\right|_{\substack{P=500\\ V=1000}} = ?$

$\frac{d}{dt}\left[PV=C\right]$

$\frac{dP}{dt}V+P\frac{dV}{dt} = 0$

$V\frac{dP}{dt} = -P\frac{dV}{dt}$ $\quad \frac{dP}{dt} = -\frac{P}{V}\frac{dV}{dt}$

$\left.\frac{dP}{dt}\right|_{\substack{P=500\\ V=1000}} = -\frac{500}{1000}\left.\frac{dV}{dt}\right|_{\substack{P=500\\ V=1000}} = -\frac{1}{2}(-20) = +10 \quad = \boxed{10\,\frac{\text{kPA}}{\text{min}}}$

quotient units: $\frac{\text{kPA}}{\text{min}}$

(this assumes we used consistent units throughout the calculation)

an extra check would be to keep the units in each step to make sure they come out right at the end