MAT1500-04/06 04F Quiz 5 Print Name (Last, First) ____________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① a) $y = \dfrac{x^2 - 2\sqrt{x}}{x}$ Evaluate $\dfrac{dy}{dx}$ not using the quotient rule.

b) Evaluate dy/dx when $x = 1$.

② $f(x) = \dfrac{9(1+x^2)}{2+x}$ a) Evaluate $f'(x)$ showing each step, one step at a time, then simplify.

b) Write an equation for the tangent line to $y = f(x)$ at $x=1$ and simplify to slope intercept form $y = mx+b$.

c) Graph f and this tangent line in an appropriate window and sketch what you see. Does it look right?

③ If $f(3) = 4$, $g(3) = 2$, $f'(3) = 6$, $g'(3) = 5$, evaluate $(fg)'(3)$.

④ $s = t^3 - 9t^2 + 15t + 10$ a) evaluate the velocity v.

b) When is the particle whose position is given by this function at rest?

c) When is the particle moving in the positive s direction? In the negative s direction?

d) What is the total distance traveled for $0 \le t \le 8$?

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① a) $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{x^2 - 2\sqrt{x}}{x}\right) = \underbrace{\dfrac{d}{dx}\left(\dfrac{x^2}{x} - \dfrac{2x^{1/2}}{x}\right)}_{\text{rewrite rule}} = \underbrace{\dfrac{d}{dx}\left(x - 2x^{-1/2}\right)}_{\text{divide thru} \quad \text{simplify}} = \dfrac{d}{dx}(x) - 2\dfrac{d}{dx}(x^{-1/2})$

diff rule, const multiplier rule

$= \underbrace{1}_{\text{identity rule}} - \underbrace{2(-\tfrac{1}{2}x^{-3/2})}_{\text{power rule}} = \underbrace{\boxed{1 + x^{-3/2}}}_{\text{simplify}}$

b) $\left.\dfrac{dy}{dx}\right|_{x=1} = 1 + 1^{-3/2} = 1 + 1 = \boxed{2}$

②a) $f'(x) = \dfrac{d}{dx}\left(\dfrac{9(1+x^2)}{2+x}\right) = 9\,\dfrac{d}{dx}\left(\dfrac{1+x^2}{2+x}\right) = 9 \cdot \dfrac{(2+x)\frac{d}{dx}(1+x^2) - (1+x^2)\frac{d}{dx}(2+x)}{(2+x)^2}$

constant multiplier rule (not necessary) quotient rule

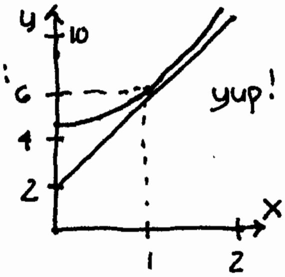
$= 9\dfrac{[(2+x)(0+2x) - (1+x^2)(0+1)]}{(2+x)^2} = \dfrac{9(4x + 2x^2 - 1 - x^2)}{(2+x)^2} = \boxed{\dfrac{9(x^2+4x-1)}{(2+x)^2}}$

sum rule, const function rule, power rule (twice)

b) $f(1) = \dfrac{9(1+1^2)}{2+1} = 6$ $f'(1) = \dfrac{9(1+4-1)}{(2+1)^2} = 4$ pt $(1,6)$ slope 4:

$y - 6 = 4(x-1) \rightarrow y = 6 + 4x - 4 = 2 + 4x$ $\boxed{y = 2+4x}$

c)



yup!

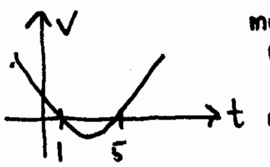
③ $(fg)'(x) = \dfrac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

$(fg)'(3) = f'(3)g(3) + f(3)g'(3) = 6(2) + 4(5) = 12 + 20 = \boxed{32}$

④ a) $s'(t) = \dfrac{d}{dt}(t^3 - 9t^2 + 15t + 10) = 3t^2 - 9(2t) + 15(1) + 0 = \boxed{3(t^2 - 6t + 5)} = \boxed{3(t-1)(t-5)}$

b) $0 = s'(t) \rightarrow \boxed{t=1 \text{ or } t=5}$

c)



moving in + direction when $t<1$ or $t>5$ since $v>0$

moving in − direction when $1<t<5$ since $v<0$

$s(0) = 10$

$s(1) = 1 - 9 + 15 + 10 = 17$ } forward 7

$s(5) = 5^3 - 9 \cdot 5^2 + 15 \cdot 5 + 10 = -15$ } backwards 32

$s(8) = \cdots = 66$ } forwards 81

$D = |s(1) - s(0)| + |s(5) - s(1)| + |s(8) - s(5)|$

$= |17 - 10| + |-15 - 17| + |66 + 15| = 7 + 32 + 81 = \boxed{120}$