MAT1500-04/06 04F Quiz 4 Print Name (Last, First) _____ 10

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① Evaluate the limit symbolically, explaining or indicating the steps in your reasoning

a) $\lim_{x\to\infty} \frac{x^2-4x}{x^2-3x-4}$ b) $\lim_{x\to -1^+} \frac{x^2-4x}{x^2-3x-4}$ (notice that both numerator and denominator factor easily)

numerical or graphical approaches, while helpful to confirm or suggest your result, are not acceptable here.

② The gravitational force exerted by Earth on a unit mass at a distance r from the center of the planet is:

$$F(r) = \begin{cases} \frac{GMr}{R^3}, & r<R \\ \frac{GM}{r^2}, & r\geq R \end{cases}$$

where M is the mass of the Earth, R is its radius, and G is the gravitational constant.

a) Write down the condition F must satisfy to be continuous at $r=R$.

b) Evaluate both sides of this equation.

c) Is F continuous at $r=R$?

① a) $\lim_{x\to\infty} \frac{x^2-4x}{x^2-3x-4} = \lim_{x\to\infty} \frac{x^2-4x}{x^2-3x-4}\cdot\frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{1-4/x}{1-3/x-4/x^2} = \frac{1}{1} = \boxed{1}$ ($4/x \to 0$, $3/x \to 0$, $4/x^2 \to 0$)

or drop lower powers: $= \lim_{x\to\infty} \frac{x^2-4x}{x^2-3x-4} = \lim_{x\to\infty} \frac{x^2}{x^2} = \lim_{x\to\infty} 1 = 1$

b) $\lim_{x\to -1^+} \frac{x^2-4x}{x^2-3x-4} = \lim_{x\to -1^+} \frac{x(x-4)^1}{(x+1)(x-4)^1} = \lim_{x\to -1^+} \frac{x}{x+1}$ ($x \to -1$, $x+1 \to 0$) $= \boxed{-\infty}$ ← division of nonzero number by zero

if $x>-1$, $x+1>0$; overall sign of expression is negative

② a) $\lim_{r\to R} F(r) = F(R)$ (functional value must agree with limiting nearby values)

b) $\lim_{r\to R^-} F(r) = \lim_{r\to R^-} \frac{GMr}{R^3} = \frac{GMR}{R^3} = \frac{GM}{R^2}$

$\lim_{r\to R^+} F(r) = \lim_{r\to R^+} \frac{GM}{r^2} = \frac{GM}{R^2}$

$\therefore \lim_{r\to R} F(r) = \frac{GM}{R^2} = F(R)$ ✓

c) Yes!

NOTES: 1-2-3 continu-i-ty!

$\lim_{x\to a} f(x) = f(a)$

(1) exists (3) equal (2) exists

$\lim_{r\to R} F(r) = F(R)$

plug in constant R for variable r (not r for R)

no "solving activity" here, only evaluation (plugging in)

missing expression

$\lim_{x\to -1^+} = 1$

do not use this private shorthand to summarize a limit calculation. use standard math notation