MAT1500-04/06 04F Quiz 3 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

① $\lim_{x \to 0^+} x \ln(x+x^2)$ $\quad x = [1.0, 0.5, 0.1, 0.05, 0.01, 0.005, 0.001]$

a) Guess the value of the limit (if it exists) by evaluating the function at the given numbers correct to six decimal places (make a table of your results).

b) Does a plot seem to confirm your guess? Make a rough sketch (with tickmarks labeled) of what you see.

[c) OPTIONAL: what is the domain of this function?]

② If an arrow is shot upward on the moon with a velocity of 58 m/s, its height y in meters after t seconds is given by $y = 58t - 0.83t^2$.

a) Find a formula for the average velocity over the time interval from $t=1$ to $t=1+h$.

b) Now evaluate it for $h = [1.0, 0.5, 0.1, 0.01, 0.001]$ (make a table of your results).

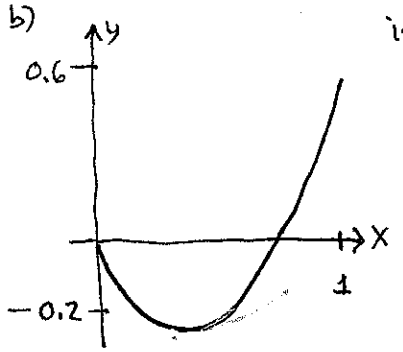
c) Guess the instantaneous velocity at $t = 1$. (units!)

①a) $f(x) = x \ln(x+x^2)$

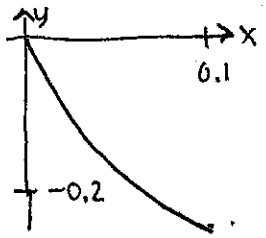
x	f(x)
1.0	0.693147
0.5	-0.143841
0.1	-0.220727
0.05	-0.147347
0.01	-0.045952
0.005	-0.026467
0.001	-0.006906

Looks like $\boxed{0}$ is the limit.

b)



if we zoom in:



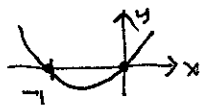
still looks convincing

Since we are interested in the limit as $x \to 0^+$, we want to plot very near the origin!

c) "ln" domain: $x + x^2 > 0$

$x(x+1) > 0$

plot $y = x(x+1)$:



clearly positive when

$\boxed{x > 0 \text{ or } x < -1}$

or $\boxed{(-\infty, -1) \cup (0, \infty)}$

② a) $V_{avg} = \dfrac{y(1+h) - y(1)}{h} = \boxed{\dfrac{58(1+h) - 0.83(1+h)^2 - [58(1) - 0.83(1)^2]}{h}}$

b)

h	V_{avg}
1.0	55.51
0.5	55.925
0.1	56.257
0.01	56.332
0.001	56.339

c) Looks like about $\boxed{56.34 \text{ m/s}}$

[NOTE: Had we simplified our expression we would have found:

$V_{avg} = \dfrac{58 + 58h - 0.83(1+2h+h^2) - 58 + 0.83}{h}$

$= \dfrac{58h - 1.66h - 0.83h^2}{h} = \dfrac{h(58 - 1.66 - 0.83h)}{h} = 56.34 - 0.83h$

so clearly the instantaneous velocity is 56.34]