MAT1500-04/06 04F Quiz 2 Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $n = f(t) = 100 \cdot 2^{t/3}$.

a) Find the inverse of this function and explain its meaning in a complete English sentence.

b) When will the population reach 50,000? Give the exact formula and the decimal approximation to 3 significant figures, with units.

c) Make a completely labeled graph that illustrates all of this information.

a) $n = 100 \cdot 2^{t/3} = f(t)$

$\ln\left[\frac{n}{100} = 2^{t/3}\right]$

$\ln\left(\frac{n}{100}\right) = \ln 2^{t/3} = \frac{t}{3}\ln 2$

$t = \boxed{\frac{3}{\ln 2}\ln\left(\frac{n}{100}\right) = f^{-1}(n)}$

This gives the number of hours it takes for the population to reach the value n.

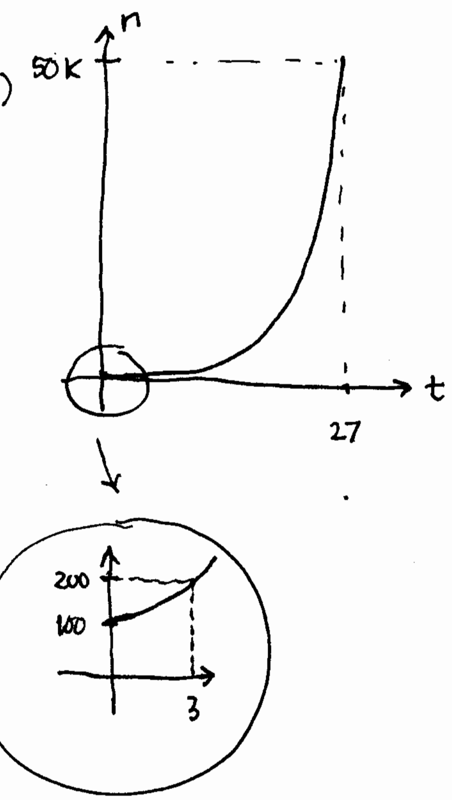
b) $f^{-1}(50{,}000) = \frac{3}{\ln 2}\ln\left(\frac{50{,}000}{100}\right)$

$= \boxed{\frac{3}{\ln 2}\ln 500}$

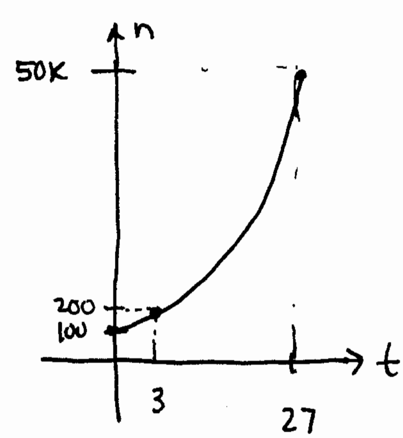
$\approx 26.897 \rightsquigarrow \boxed{26.9 \text{ hours}}$

[NOTE! once you have the inverse function there is no need to resolve the original equation for a particular n value like 50,000!!]

c)



one plot cannot show the small scale starting behavior and the large scale growth UNLESS we distort the graph to make a sort of "caricature":



NOTE: In solving for the inverse function it makes no sense to interchange the variable names in the equation because each carries a distinct meaning with different units.