MAT1500-04/06 04F Test 3 Takehome Answers

① $y = \ln(\arcsin(\frac{x}{2}))$

$$\frac{dy}{dx} = \frac{1}{\arcsin(\frac{x}{2})} \underbrace{\frac{d}{dx}\arcsin(\tfrac{x}{2})}_{\frac{1}{\sqrt{1-(\frac{x}{2})^2}}\underbrace{\frac{d}{dx}\left(\frac{x}{2}\right)}_{\frac{1}{2}}} = \boxed{\frac{1}{2\arcsin(\frac{x}{2})}\frac{1}{\sqrt{1-x^2/4}}}$$

$30°$, $\frac{1}{2}$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{2\arcsin(\frac{1}{2})}\frac{1}{\sqrt{1-1/4}} = \frac{1}{2(\frac{\pi}{6})\sqrt{\frac{3}{4}}} = \boxed{\frac{6}{\pi\sqrt{3}} = \frac{2\sqrt{3}}{\pi}}$$

② $y = 15(e^{x/30} + e^{-x/30}) - 25$

$$\frac{dy}{dx} = 15\left(e^{\frac{x}{30}}\left(\tfrac{1}{30}\right) + e^{-\frac{x}{30}}\left(-\tfrac{1}{30}\right)\right) - 0 = \tfrac{1}{2}\left(e^{\frac{x}{30}} - e^{-\frac{x}{30}}\right)$$

$$\left.\frac{dy}{dx}\right|_{x=6} = \tfrac{1}{2}\left(e^{\frac{6}{30}} - e^{-\frac{6}{30}}\right) = \boxed{\tfrac{1}{2}\left(e^{1/5} - e^{-1/5}\right) \approx 0.201}$$

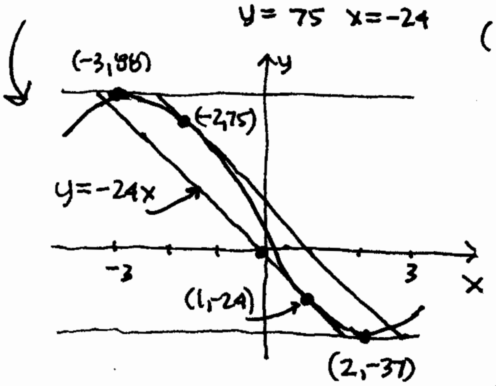
③ $y = 2x^3 + 3x^2 - 36x + 7$

$$\frac{dy}{dx} = 6x^2 + 6x - 36 = 6(x^2+x-6) = 6(x+3)(x-2)$$

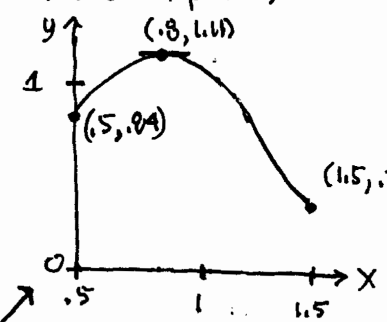
$\overset{(a)}{=} 0 \to x = -3,\ x = 2$; $y = 88$, $y = -37$ $\quad\boxed{(-3,88),\ (2,-37)}$

$\overset{(b)}{=} -24 \xrightarrow{(\div 6)} x^2+x-6 = -4 \to x^2+x-2 = (x+2)(x-1) = 0$

$x = -2,\ x = 1$; $y = 75$, $x = -24$ $\quad\boxed{(-2,75),\ (1,-24)}$



($y=-24x$ is the tangent line to the last point)



④ $f(x) = x^2(2-x)^3$

$$f'(x) = \frac{d}{dx}(x^2)\,(2-x)^3 + x^2\frac{d}{dx}(2-x)^3 = (2x)(2-x)^3 + x^2\cdot 3(2-x)^2(0-1)$$

$$= x(2-x)^2[\underbrace{2(2-x) - 3x}_{4-5x}] = x(2-x)^2(4-5x) = 0$$

$x=0$, $x=2$: outside; $x = \frac{4}{5} = 0.8$: one critical # inside

$$f(\tfrac{4}{5}) = \left(\tfrac{4}{5}\right)^2\left(2-\tfrac{4}{5}\right)^3 = \left(\tfrac{4}{5}\right)^2\left(\tfrac{6}{5}\right)^3 = \frac{2^{10}\cdot 3^3}{5^5} = \frac{3456}{3125} \approx 1.11 \to \boxed{\text{abs max } \left(\tfrac{4}{5}, \tfrac{3456}{3125}\right)} \approx (.8, 1.11)$$

$$f(\tfrac{1}{2}) = \left(\tfrac{1}{2}\right)^2\left(2-\tfrac{1}{2}\right)^3 = \frac{1}{2^2}\left(\frac{3}{2}\right)^3 = \frac{27}{32} \approx 0.84$$

$\approx (.5, .28)$

$$f(\tfrac{3}{2}) = \left(\tfrac{3}{2}\right)^2\left(2-\tfrac{3}{2}\right)^3 = \frac{3^2}{2^2}\left(\frac{1}{2}\right)^3 = \frac{9}{32} \approx 0.28 \to \boxed{\text{abs. min } \left(\tfrac{3}{2}, \tfrac{9}{32}\right)}$$

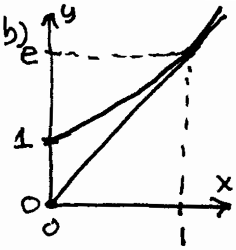
⑤ a) $y = e^x \quad \frac{dy}{dx} = e^x \quad x = a \to y = e^a \to \frac{dy}{dx} = e^a$

$y - e^a = e^a(x-a) \to y = e^a + e^a(x-a) = e^a(x+1-a)$

$(x,y) = (0,0)$: $0 = e^a(0+1-a) \to 1-a = 0 \to a = 1$

so $\boxed{y = e^1(x+1-1) = ex} = L(x)$, c) $L(1.1) = e(1.1) \approx \boxed{2.990 \approx e^{1.1}}$

⑤ b)



notice the linear approx should be a bit low

in fact $e^{1.1} \approx 3.004$

but the error is less than half a percent

⑥ a) $\frac{d}{dx}[x^2 + 4xy + y^2 = 13]$

$$\underbrace{\frac{d}{dx}(x^2)}_{2x} + 4\underbrace{\frac{d}{dx}(xy)}_{\underbrace{\frac{dx}{dx}}_{1}y + x\frac{dy}{dx}} + \underbrace{\frac{d}{dx}y^2}_{2y\frac{dy}{dx}} = \underbrace{\frac{d}{dx}(13)}_{0}$$

$$2x + 4\left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$2x + 4y + (4x+2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{(2x+4y)}{4x+2y} = -\frac{(x+2y)}{(2x+y)}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=2\\y=1}} = -\frac{(2+2\cdot 1)}{(2\cdot 2+1)} = -\frac{4}{5}$$

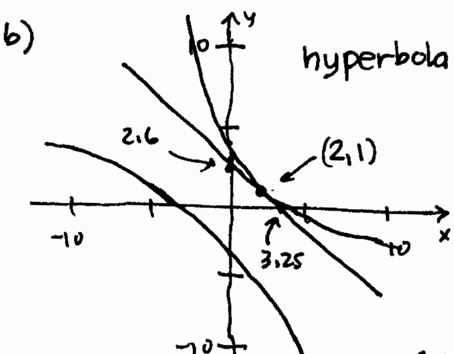
$y - 1 = -\frac{4}{5}(x-2)$, $y = 1 - \frac{4}{5}(x-2) = \frac{5-4x+8}{5}$

$\boxed{y = \frac{13-4x}{5}}$

$x = 0 \to \boxed{y = \frac{13}{5} = 2.6 \quad y \text{ intercept}}$

$y = 0 \to \boxed{x = \frac{13}{4} = 3.25 \quad x \text{ intercept}}$

b) hyperbola



⑦ a) $y = \frac{\sin(mx)}{x}$

$$\frac{dy}{dx} = \frac{x\frac{d}{dx}\sin(mx) - \sin(mx)\underbrace{\frac{d}{dx}x}_{1}}{x^2}$$

$\cos(mx)\underbrace{\frac{d}{dx}(mx)}_{m}$

$$= \boxed{\frac{mx\cos(mx) - \sin(mx)}{x^2}} \overset{(b)}{=} 0$$

$\to \boxed{mx\cos(mx) - \sin(mx) = 0}$ or:

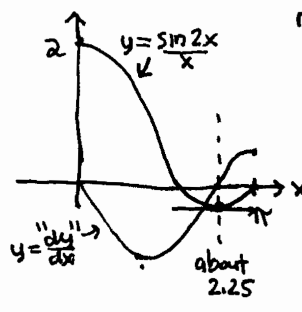
$\div \cos mx$

$\boxed{mx - \tan(mx) = 0}$

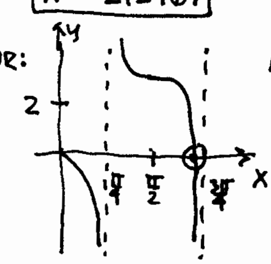
c) $m = 2$: $2x - \tan 2x = 0$

root between 2 and 2.5:

$\boxed{x \approx 2.2467}$

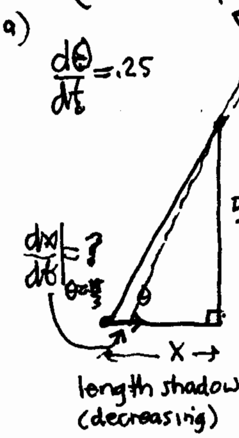


OR:



(8) (horizon to horizon ~ $180° \to \pi$ in a half day ~ 12 hr } $\frac{\pi}{12} \approx 0.26 \frac{rad}{hr}$)

a)



b) $\frac{x}{50} = \cot\theta \to \boxed{x = 50\cot\theta}$

Given: $\frac{d\theta}{dt} = .25 \frac{rad}{hr}$

Goal: $\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{3}} = ?$

$\frac{\sqrt{3}}{2} = \sin\frac{\pi}{6}$

c) $\frac{d}{dt}\left[x = 50\cot\theta\right]$

$\frac{dx}{dt} = 50(-\csc^2\theta)\frac{d\theta}{dt}$

$\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{3}} = -\frac{50}{\sin^2\frac{\pi}{3}}(.25) = -\frac{50}{\left(\frac{\sqrt{3}}{2}\right)^2}.25$

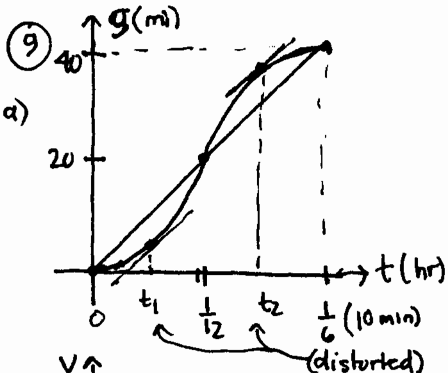
$= -\frac{50}{3} = -16\frac{2}{3} \approx -16.7$ ft/hr

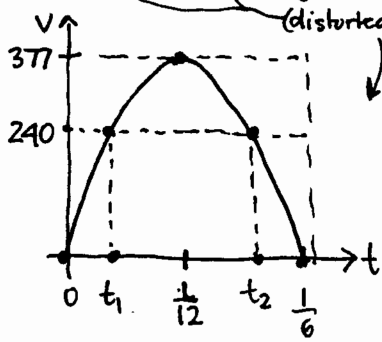
d) The length of the shadow is decreasing at about 16.7 ft/hr when the elevation of the sun is 60°.

e) $\left(-\frac{50}{3}\frac{ft}{hr}\right)\left(\frac{12\,in}{ft}\right)\left(\frac{1\,hr}{60\,min}\right) = -\frac{10}{3}\frac{in}{min} = \boxed{-3\tfrac{1}{3}\ in/min \approx -3.3\ in/min}$

(note: this is just the differential approximation: $dx = \left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{3}} dt$)

(9) a)





b) $\frac{\Delta s}{\Delta t} = \frac{40\ mi}{\frac{1}{6}\ hr} = \boxed{240\ mph}$

c) $s = 20(1-\cos(6\pi t))$

$\boxed{v = \frac{ds}{dt} = 20\left(0 - \frac{d}{dt}\cos(6\pi t)\right) = 120\pi\sin(6\pi t)}$ $= -20(-\sin 6\pi t)\underbrace{\frac{d}{dt}(6\pi t)}_{6\pi}$

$v|_{t=\frac{1}{12}} = 120\pi\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} \approx 377$

d) $v = 240 \to 120\pi\sin(6\pi t) = 240 \to \sin(6\pi t) = \frac{2}{\pi}$

$t_1 = \frac{1}{6\pi}\arcsin\frac{2}{\pi} \approx .0367\ hr \approx \boxed{2.20\ min}$

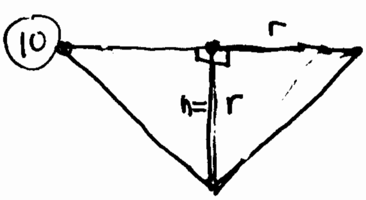
$t_2 = \frac{1}{6} - t_1 \approx .0.1301\ hr \approx \boxed{7.80\ min}$

e) The peak velocity occurs at midtrip: $t = \frac{1}{12}$ hr $= 5$ min

f) $a = \frac{dv}{dt} = \frac{d}{dt}(120\pi\sin(6\pi t)) = (120\pi)\cos(6\pi t)\underbrace{\frac{d}{dt}(6\pi t)}_{6\pi} = \boxed{720\pi^2\cos(6\pi t)}$

Acceleration/deacceleration is the slope of v versus t, so slope steepest at beginning and end of trip: $|a|_{max} = \boxed{720\pi^2\ mi/hr^2 \approx 7106\ mi/hr \approx 118.4\ \frac{mph}{min}}$ (more appropriate units for interpretation)

(10)



$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$

$\frac{dV}{dr} = \frac{1}{3}\pi(3r^2) = \pi r^2$

$dV = \pi r^2 dr$

$\frac{dV}{V} = \frac{\pi r^2 dr}{\frac{1}{3}\pi r^3} = 3\frac{dr}{r}$

$dr = .02r$ or $\frac{dr}{r} = .02$ (2% increase)

$\frac{dV}{V} = 3(.02) = .06 \to \boxed{6\%\ increase}$