MAT1500-04/06 04F Test 2 Answers

① $f(x) = \dfrac{e^{-2x}}{1+x^2}$ a) $f'(x) = \dfrac{d}{dx}\left(\dfrac{e^{-2x}}{1+x^2}\right) = \dfrac{(1+x^2)\frac{d}{dx}(e^{-2x}) - e^{-2x}\frac{d}{dx}(1+x^2)}{(1+x^2)^2} = \dfrac{(1+x^2)(e^{-2x}(-2)) - e^{-2x}(0+2x)}{(1+x^2)^2}$

$= \dfrac{(-2-2x^2-2x)e^{-2x}}{(1+x^2)^2} = \boxed{\dfrac{-2(1+x+x^2)}{(1+x^2)^2}e^{-2x}}$

b) $f'(x) = 0 \rightarrow 1+x+x^2 = 0 \rightarrow x = \dfrac{-1\pm\sqrt{1-4(1)(1)}}{2} = -\dfrac{1\pm\sqrt{3}\,i}{2}$ no real solutions.

$\boxed{\text{so tangent line is never horizontal}}$ (In fact $f'(x) < 0$ so function always decreasing.)

② a) $y = x^2\cos(2\pi x)$ $\quad \dfrac{dy}{dx} = \dfrac{d}{dx}(x^2\cos(2\pi x)) = \underbrace{\dfrac{d}{dx}(x^2)}_{2x}\cos(2\pi x) + x^2\underbrace{\dfrac{d}{dx}(\cos(2\pi x))}_{-\sin(2\pi x)\underbrace{\frac{d}{dx}(2\pi x)}_{2\pi}}$

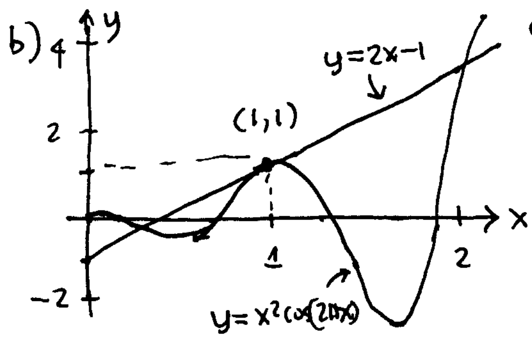
$= 2x\cos(2\pi x) - 2\pi x^2\sin(2\pi x)$

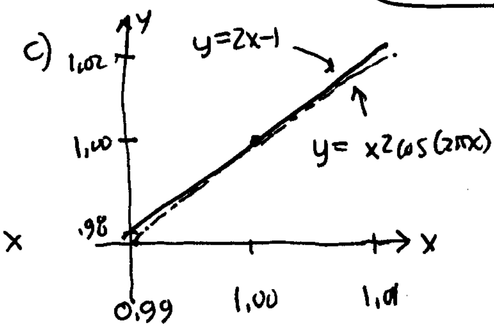
$\left.\dfrac{dy}{dx}\right|_{x=1} = 2\cdot1\cdot\underbrace{\cos(2\pi)}_{1} - 2\pi(1)^2\underbrace{\sin(2\pi)}_{0} = 2.$

$y|_{x=1} = 1^2\cos(2\pi) = 1$

$y-1 = 2(x-1) = 2x-2$

$\boxed{y = 2x-2+1 = 2x-1}$

b) 

c) 

yes, the curve approaches the tangent line. one more power of 10 zoom and the pixels merge.

caution: If you put instead a slope 1 line through this point: $y-1 = 1(x-1) \rightarrow y = x$ it will still "look tangent" in the first window scale, but when you zoom in you will see the clear crossing of the two curves — not tangent.

③ a) $s = Ae^{-ct}\sin(\omega t+\delta)$ $\quad v = \dfrac{ds}{dt} = \dfrac{d}{dt}Ae^{-ct}\sin(\omega t+\delta) = A\dfrac{d}{dt}e^{-ct}\sin(\omega t+\delta)$

$= A\Big(\underbrace{\dfrac{d}{dt}(e^{-ct})}_{e^{-ct}\underbrace{\frac{d}{dt}(-ct)}_{-c}}\sin(\omega t+\delta) + e^{-ct}\underbrace{\dfrac{d}{dt}(\sin(\omega t+\delta))}_{\cos(\omega t+\delta)\underbrace{\frac{d}{dt}(\omega t+\delta)}_{\omega+0}}\Big) = \boxed{Ae^{-ct}\underbrace{(-c\sin(\omega t+\delta)+\omega\cos(\omega t+\delta))}_{=0}}$

$\dfrac{c\sin(\omega t+\delta)}{\cos(\omega t+\delta)} = \omega\dfrac{\cos(\omega t+\delta)}{\cos(\omega t+\delta)}$

$\rightarrow c\tan(\omega t+\delta) = \omega \rightarrow \boxed{\tan(\omega t+\delta) = \dfrac{\omega}{c}}$

④

	t	L		
Δt	307	10.059		
1			$-.035$	ΔL
	308	10.024		
1			$-.033$	
	309	9.991		

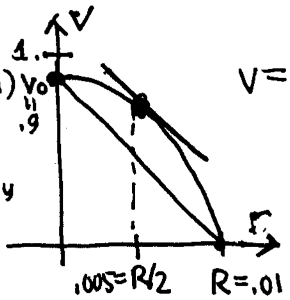
$\dfrac{\Delta L}{\Delta t} = -.035$

$\dfrac{\Delta L}{\Delta t} = -.033$

avg $= \frac{1}{2}(-.035 + -0.33) = -0.034\ \dfrac{hr}{day}$

The length of day is decreasing at a rate of 0.034 hr/day

or $\dfrac{0.034\,hr}{day}\ \dfrac{60\,min}{hr} = 2.04\ \dfrac{min}{day}$.

⑤ a) 

no technology needed

$v = .9\left(1 - \dfrac{r^2}{.01^2}\right)$

b) $(0, v_0)$ to $(R, 0)$: $\dfrac{\Delta v}{\Delta R} = \dfrac{0-v_0}{R-0} = \dfrac{-v_0}{R} = -\dfrac{.9}{.01}$

$\boxed{-90\ \dfrac{cm/sec}{cm}}$

c) $\dfrac{dv}{dr} = \dfrac{d}{dr}\left(v_0\left(1-\dfrac{r^2}{R^2}\right)\right)$

$= v_0\left(0 - \dfrac{2r}{R^2}\right) = -\dfrac{2v_0 r}{R^2}$ $\quad \left.\dfrac{dv}{dr}\right|_{r=\frac{R}{2}} = \dfrac{-2v_0 R/2}{R^2} = -\dfrac{v_0}{R}$

or $\dfrac{dv}{dr} = \dfrac{d}{dr}\left(.9\left(1-\dfrac{r^2}{.01^2}\right)\right) = .9\left(0 - \dfrac{2r}{(.01)^2}\right) = -\dfrac{1.8r}{(.01)^2}$, $\left.\dfrac{dv}{dr}\right|_{r=.005} = \dfrac{-1.8(.005)}{(.01)^2} = -\dfrac{.9}{.01} = -90$

$\boxed{-90\ \dfrac{cm/sec}{cm}}$ same as avg.

d) secant line connects points $(0, v_0)$ and $(R, 0)$

tangent line is parallel (redraw graph if necessary) to make it look right