Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers exact (no decimal approximations, if possible). [See long instructions on reverse].

① $f(x) = \dfrac{e^{-2x}}{1+x^2}$ a) Evaluate $f'(x)$ and simplify to its simplest factored form.
   b) Does the graph of f have any horizontal tangent lines? Support your response with a calculation.

② $y = x^2 \cos(2\pi x)$ a) Write an equation for the tangent line to the graph of this function at $x=1$ and simplify to slope-intercept form.
   b) Make a completely labeled sketch of what you see in an appropriate technology window of a plot showing both this function and this tangent line.
   c) Does it look right when you zoom in? Explain.

③ An equation of motion of the form $s = A e^{-ct} \sin(\omega t + \delta)$, where $A>0$, $c>0$, $\omega>0$ and δ are all constants, represents the damped oscillation of an object (in time). a) If s is the position, find the velocity v and factor out any common factors from your result when you simplify it.
   b) Only one of these factors can be zero. Use its vanishing to obtain a condition on $\tan(\omega t + \delta)$ which corresponds to a time when the object is momentarily at rest.

④

t	L
305	10.129
306	10.094
307	10.059
308	10.024
309	9.99♦
310	9.957

According to the model for the length of day in Philly, in a normal 365 day year, the length of day for the first 6 days of November are given in the table (units of hours).
According to this table of values, what is the time rate of change of the length of day on November 4 in such a year?
Answer in a complete English sentence, and also give the equivalent result in minutes per day.

⑤ $V(r) = V_0(1 - r^2/R^2)$, $0 \le r \le R$, $V_0 = 0.9$ cm/s , $R = .01$ cm
   a) Make a rough diagram of V versus r (labeling key tickmarks).
   b) What is the average rate of change of V with respect to r as r increases from 0 to R? (This is called the average velocity gradient.)
   c) What is the instantaneous rate of change of V with respect to r at $r = R/2$? (This is called the velocity gradient.)
   d) Add a secant line to your diagram corresponding to the average rate of change in b) and a piece of the tangent line of c).