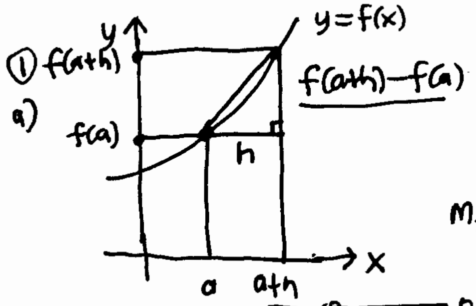
MAT1500-04/06 04F Test 1B Answers

① a)



$m_{sec} = \dfrac{f(a+h)-f(a)}{h}$ $\quad m_{tan} = \lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

b) $f(x) = \dfrac{2}{x-1}$ $\quad f(a) = \dfrac{2}{a-1}$ $\quad f(a+h) = \dfrac{2}{(a+h)-1}$

$m_{tan} = \lim\limits_{h\to 0} \dfrac{\dfrac{2}{a+h-1} - \dfrac{2}{a-1}}{h} = \lim\limits_{h\to 0} \dfrac{\dfrac{2(a-1)-2(a+h-1)}{(a+h-1)(a-1)}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{2a}-\cancel{2}-\cancel{2a}-2h+\cancel{2}}{(a+h-1)(a-1)h} = \lim\limits_{h\to 0} \dfrac{-2\cancel{h}}{\cancel{h}(a+h-1)(a-1)} = \lim\limits_{h\to 0} \dfrac{-2}{(a+h-1)(a-1)} = \dfrac{-2}{(a+0-1)(a-1)}$

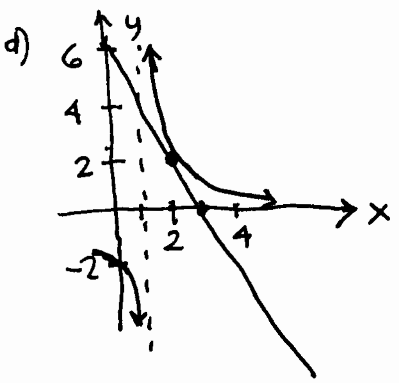
$= \boxed{\dfrac{-2}{(a-1)^2}}$ c) $x=2 \to y=f(2) = \dfrac{2}{2-1} = \dfrac{2}{1} = 2$ $\quad m_{tan} = \dfrac{-2}{(2-1)^2} = \dfrac{-2}{1^2} = \dfrac{-2}{1} = -2$

pt $(2,2)$, slope -2, pt-slope: $y-2 = -2(x-2) = -2x+4$, $\boxed{y=-2x+6}$

x-intercept: $y=0 \to 0=-2x+6 \to \boxed{x=3}$

y-intercept: $x=0 \to \boxed{y = -2(0)+6 = 6}$

d)



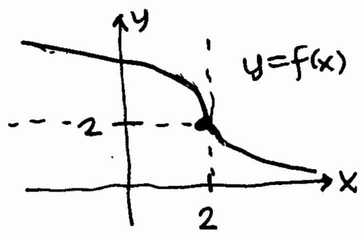
② a) $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} \dfrac{2}{x-1} = \dfrac{2}{2-1} = 2$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} 2+\sqrt{2-x} = 2+\sqrt{2-2} = 2$

$\Big\}$ $\lim\limits_{x\to 2} f(x) = 2 = f(2)$ so continuous at $x=2$.

But $\dfrac{2}{x-1}$ has problems only at $x=1$ where it does not apply and $2+\sqrt{2-x}$ has problems (complex) only when $x>2$ which doesn't apply, so the function is continuous everywhere else and so is continuous on the entire real line.

②b)



③ a) (i) $\lim\limits_{x\to\infty} \dfrac{4-x\ /x}{3+x\ /x} = \lim\limits_{x\to\infty} \dfrac{\frac{4}{x}^{\to 0}-1}{\frac{3}{x}_{\to 0}+1} = \dfrac{-1}{1} = \boxed{-1}$ $\overset{(ii)}{=} \lim\limits_{x\to-\infty} \dfrac{4-x}{3+x}$ same calculation.

(iii) $\lim\limits_{x\to -3^+} \dfrac{4-x \to 7}{3+x \to 0^+} = \boxed{+\infty}$ overall sign + (>0)

(iv) $\lim\limits_{x\to -3^-} \dfrac{4-x \to 7}{3+x \to 0^-} = \boxed{-\infty}$ overall sign negative (<0)

③(v) $\lim\limits_{x\to -3} \dfrac{4-x}{3+x}$ does not exist. (one-sided limits do not agree)

(b) Since $\lim\limits_{x\to\pm\infty} f(x) = -1$, $\boxed{y=-1}$ is a hor. asymptote in both directions

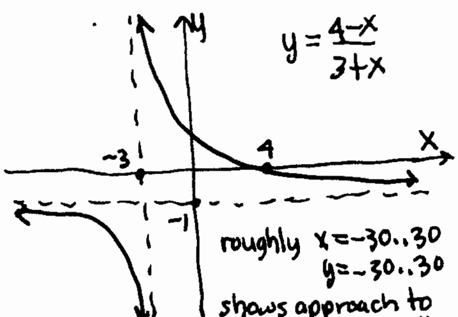
(c) Since $\lim\limits_{x\to -3^\pm} f(x) = \pm\infty$, $\boxed{x=-3}$ is a vertical asymptote

(d) $\dfrac{4-x}{3+x} = 0 \to 4-x=0 \to \boxed{x=4}$

e) $\dfrac{4-x}{3+x} = -1 \to 4-x = -(3+x) = -3-x \to$ (add $-3-x$ to both sides) $7=0$, not possible. $\boxed{\text{no solution}}$

(so graph does not cross line $y=-1$)

(f)



roughly $x=-30..30$, $y=-30..30$ shows approach to asymptotes well

④ a) $\lim\limits_{x\to 0} \dfrac{1}{2+e^{-3/x^2}}$ $\left\{ \dfrac{3}{x^2} \to \dfrac{3}{0^+} = +\infty,\ e^{-\infty} = 0 \right.$ decaying exponential

$= \dfrac{1}{2+0} = \boxed{\dfrac{1}{2}}$

b) $\lim\limits_{x\to\infty} \dfrac{1}{2+e^{-3/x^2}} = \dfrac{1}{2+e^0} = \dfrac{1}{2+1} = \boxed{\dfrac{1}{3}}$ $\left\{ \dfrac{3}{\infty^2} = 0 \right.$ reciprocal powers of x go to zero as $x\to\pm\infty$

c) all we needed was that 3 was positive, so yes, it would make no difference, same results.