

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers exact (no decimal approximations, if possible). [See long instructions on reverse].

① A lake is stocked with 500 fish and their population increases according to the rule  $P = f(t) = \frac{10,000}{1 + 19 e^{-t/5}}$ , where  $t$  is the time measured in days/months.

- a) Evaluate the limiting population  $\lim_{t \rightarrow \infty} P$ .
- b) Express  $t$  as a function of  $P$  and use your result to see how long it takes for the fish population to double to 1000 (think units and significant digits).

②  $f(x) = \begin{cases} \sqrt{x-5}, & x \neq 25 \\ \frac{1}{5}, & x = 25 \end{cases}$  Is this function continuous on the entire real line? Explain using limit notation and symbolically evaluating any relevant limits.

③ Consider the graph  $y = 2x - x^2$ .

- a) Find the slope of the tangent line to this graph at  $x=a$  using the limit definition.
- b) Use your result to write the equation of the tangent line at  $x=3$  and simplify it to the form  $y=ax+b$ .
- c) Use technology and an appropriate window to graph the function and this tangent line together. Make a rough sketch (with labeled tickmarks) of what you see. Does it look right? Explain.

④  $f(x) = \frac{2x^2 - 6x + 4}{x^3 - 6x^2 + 12x - 8} = \frac{2(x-1)(x-2)}{(x-2)^3}$  (the factored form of this rational function)

- a) What is the domain of  $f$ ? Explain.
- b) Does  $f$  have horizontal asymptotes? If so, write down and evaluate the appropriate limit or limits associated with these asymptotes and then write the equation of the asymptotes.
- c) Does  $f$  have vertical asymptotes? If so, write down and evaluate appropriate one-sided limits (and two-sided limits if they can be assigned a symbolic value) symbolically or by reasoning that describe how  $f$  behaves near them. Then write the equation of the asymptotes.

Remember, while a graph or numerics may be useful in suggesting the values of these limits, your evaluation of the limit symbolically or by reasoning is intended to explain why you see what you see with technology.

⑤ Explain why the Intermediate Value Thm\* guarantees that the equation  $f(x) = e^{-x} - x = 0$  has a root (solution) on the interval  $0 \leq x \leq 1$ .

\*IVT: Suppose that  $f$  is continuous on the closed interval  $[a, b]$  and let  $N$  be any number between  $f(a)$  and  $f(b)$ , where  $f(a) \neq f(b)$ . Then there exists a number  $c$  in  $(a, b)$  such that  $f(c) = N$ .