MAT2705-01/04 03S Quiz 8 Print Name (Last, First) ______________ last first

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Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations).

① $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ Find the eigenvalues of this matrix.

② $A = \begin{bmatrix} 2 & 0 & 3 \\ -3 & -1 & -3 \\ 0 & 0 & -1 \end{bmatrix}$ This matrix has eigenvalues : $\lambda = -1, -1, 2$.
Find a basis of the two eigenspaces, giving your final results in the form: $\lambda = -1 :$ [*,*,*], [*,*,*]
$\lambda = 2 :$ [*,*,*]

① $A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix} \rightarrow \det(A-\lambda I) = (\lambda-1)^2 - 4 = \lambda^2 - 2\lambda + 1 - 4$
$= \lambda^2 - 2\lambda - 3 = (\lambda+1)(\lambda-3) = 0$

$\boxed{\lambda = -1, 3}$

② $\lambda = -1 :$ $A - \lambda I = \begin{bmatrix} 2-(-1) & 0 & 3 \\ -3 & -1-(-1) & -3 \\ 0 & 0 & -1-(-1) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ -3 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad x_1 + x_3 = 0 \longrightarrow x_1 = -t_2$
L F F $\quad 0 = 0 \quad x_2 = t_1$
$\quad 0 = 0 \quad x_3 = t_2$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1 \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{\vec{b}_1} + t_2 \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}_{\vec{b}_2}$

$\lambda = 2 :$ $A - \lambda I = \begin{bmatrix} 2-2 & 0 & 3 \\ -3 & -1-2 & -3 \\ 0 & 0 & -1-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 \\ -3 & -3 & -3 \\ 0 & 0 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad x_1 + x_2 = 0 \longrightarrow x_1 = -t$
L F L $\quad x_3 = 0 \quad x_2 = t$
$\quad 0 = 0$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t \\ t \\ 0 \end{bmatrix} = t \underbrace{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}_{\vec{b}_3}$

$$\boxed{\begin{array}{ll} \underline{\text{summary}} & \lambda = -1 : \ [0,1,0], [-1,0,1] \\ & \lambda = 2 : \ [-1,1,0] \end{array}}$$