MAT2705-01/04 03S Quiz 6 Print Name (Last, First) ____________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations).

① $\vec{v}_1 = [3,9,0,5]$, $\vec{v}_2 = [3,0,9,-7]$, $\vec{v}_3 = [4,7,5,0]$

If these vectors are linearly independent, show this (and explain why your work justifies your conclusion); otherwise find a nontrivial linear combination of them which is equal to the zero vector.

Be sure to state the augmented matrix $\underset{\sim}{S}$ you use to answer this question as well as the result rref(S) you obtain with technology. If the vectors are linearly dependent, your final response should be something like $2\vec{v}_1 - \vec{v}_2 + 3\vec{v}_3 = 0$.

② $y'' - 4y = 0$; $y_1 = e^{2x}$, $y_2 = e^{-2x}$; $y(0) = 1$, $y'(0) = 0$.

Find a particular solution of the DE of the form $y = c_1 y_1 + c_2 y_2$ that satisfies the initial conditions. Then check that your result satisfies the D.E.

① $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$

$$\begin{bmatrix} 3 & 3 & 4 \\ 9 & 0 & 7 \\ 0 & 9 & 5 \\ 5 & -7 & 0 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 3 & 3 & 4 & 0 \\ 9 & 0 & 7 & 0 \\ 0 & 9 & 5 & 0 \\ 5 & -7 & 0 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{cccc} L & L & F & \\ c_1 & c_2 & c_3 & \end{array} \begin{bmatrix} 1 & 0 & 7/9 & 0 \\ 0 & 1 & 5/9 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} c_1 + 7/9\, c_3 &= 0 \\ c_2 + 5/9\, c_3 &= 0 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

$c_3 = t$ $\quad c_1 = -7/9\, t$, $\quad c_2 = -5/9\, t$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -7/9\, t \\ -5/9\, t \\ t \end{bmatrix} = t \underbrace{\begin{bmatrix} -7/9 \\ -5/9 \\ 1 \end{bmatrix}}$$

coefficients of single independent linear relationship:

$$\boxed{-7/9\,\vec{v}_1 - \tfrac{5}{9}\vec{v}_2 + \vec{v}_3 = 0}$$

CHECK: (not necessary)

$$-\frac{7}{9}\begin{bmatrix} 3 \\ 9 \\ 0 \\ 5 \end{bmatrix} - \frac{5}{9}\begin{bmatrix} 3 \\ 0 \\ 9 \\ -7 \end{bmatrix} + \begin{bmatrix} 4 \\ 7 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-21-15+36}{9} \\ \frac{-63-0+63}{9} \\ \frac{0-45+45}{9} \\ \frac{-35+35+0}{9} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

② $y = c_1 e^{2x} + c_2 e^{-2x}$

$y' = 2c_1 e^{2x} - 2c_2 e^{-2x}$

$y(0) = c_1 + c_2 = 1$

$y'(0) = 2c_1 - 2c_2 = 0$

$$\to \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/2 \end{bmatrix}$$

so $c_1 = 1/2$, $c_2 = 1/2$ and $\boxed{y = \tfrac{1}{2}e^{2x} + \tfrac{1}{2}e^{-2x} = \dfrac{e^{2x} + e^{-2x}}{2}}$ $(= \cosh 2x)$

check D.E.:

$$\begin{aligned} y' &= e^{2x} - e^{-2x} \\ y'' &= 2e^{2x} + 2e^{-2x} \\ y'' - 4y &= (2e^{2x} + 2e^{-2x}) - 4\left(\frac{e^{2x}}{2} + \frac{e^{-2x}}{2}\right) \\ &= (2-2)e^{2x} + (2-2)e^{-2x} = 0 \ \checkmark \end{aligned}$$