

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem.

$\frac{dP}{dt} = (\beta - \delta)P$  a) Suppose that the fish population  $P(t)$  in a lake is attacked by a disease at time  $t=0$ , with the result that the fish cease to reproduce (so that the birthrate is  $\beta=0$ ) and the death rate  $\delta$  (deaths per week per fish) is thereafter proportional to  $1/\sqrt{P}$ . If there were initially 900 fish in the lake and 441 were left after 6 weeks, how long did it take all the fish in the lake to die? b) Make a rough graph of  $P$  versus  $t$  for this problem: clearly labeling the  $t$  and  $P$  values of the 3 data points which describe this word problem (initial, intermediate, final).

a)  $\delta \propto 1/\sqrt{P} \rightarrow \delta = k/\sqrt{P} \rightarrow \frac{dP}{dt} = -\left(\frac{k}{P^{1/2}}\right)P = -k P^{1/2}$  (separable DE)

$\int P^{-1/2} dP = -\int k dt$  (sep & integrate)

$\frac{P^{1/2}}{1/2} = -kt + C_1 \xrightarrow{\text{solve}} P^{1/2} = \frac{1}{2} C_1 - \frac{1}{2} kt = C - \frac{1}{2} kt \geq 0$

$\downarrow$   
 $P = (C - \frac{1}{2} kt)^2$  gen. soln.

Initial condition:  $P_0^{1/2} = C - \frac{1}{2} k(0) = C$

$\boxed{P = (P_0^{1/2} - \frac{1}{2} kt)^2}$

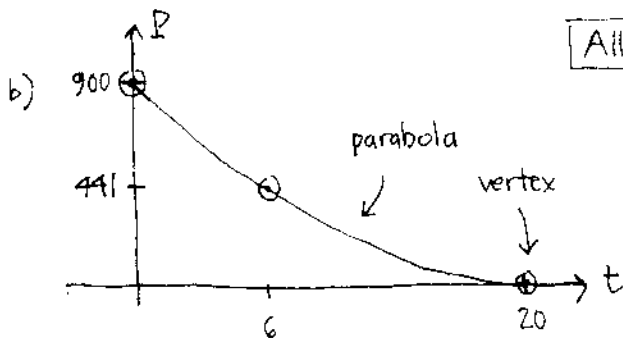
Given:  $P_0 = P(0) = 900 = 30^2 \rightarrow P = (30 - \frac{1}{2} kt)^2$   
 $P(6) = 441 = 21^2 \rightarrow P(6) = (30 - \frac{1}{2} k \cdot 6)^2 = 441 = 21^2$

$30 - 3k = 21$   
 $9 = 3k, k = 3$

$\boxed{P = (30 - \frac{3}{2} t)^2} \stackrel{?}{=} 0 \rightarrow 30 - \frac{3}{2} t = 0$

$t = \frac{2}{3}(30) = 20$

**All fish are dead after 20 weeks.**



check:  $P = (30 - \frac{3}{2} t)^2$   
 $\frac{dP}{dt} = 2(30 - \frac{3}{2} t)(-\frac{3}{2}) = -3(30 - \frac{3}{2} t)$   
 DE:  $\frac{dP}{dt} = -3P^{1/2}$   
 $-3(30 - \frac{3}{2} t) = -3((30 - \frac{3}{2} t)^2)^{1/2}$   
 $= -3(30 - \frac{3}{2} t) \checkmark$  (since  $30 - \frac{3}{2} t \geq 0$ )