MAT2705-01/04 03S Quiz 3 Print Name (Last, First) _____ 10

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem.

Last century people liked to think a 10% average annual return on investment was a good number. Solve the following IVP for the amount $A(t)$ in thousands of dollars of a retirement investment of a worker whose starting salary is 30 thousand dollars and t is the number of years worked, during which the salary $S(t) = 30e^{.05t}$ increases at a (continuous) annual rate of 5%, while 12% is put into the retirement fund:

$$\begin{cases} \frac{dA}{dt} = 0.12(30)e^{.05t} + 0.10A \\ A(0) = 0 \end{cases}$$

First express $A(40)$, the retirement value after 40 years, in dollars and cents to the nearest cent, and then round to 3 significant figures.

[EXTRA CREDIT: solve this problem with the dsolve template and evaluate $A(40)$ for 4pts extra credit. email me the worksheet with webmail to claim the credit.]

$$e^{-0.10t}\left[\frac{dA}{dt} - 0.10A = 3.6e^{.05t}\right] \rightarrow \frac{d}{dt}\left(Ae^{-0.10t}\right) = 3.6e^{-0.10t}e^{0.05t} = 3.6e^{-0.05t}$$

$$e^{\int -0.10\,dt} = e^{-0.10t}$$

$$Ae^{-0.10t} = \int 3.6e^{-0.05t}\,dt$$

$$= 3.6\frac{e^{-0.05t}}{-0.05} + C$$

$$= -\frac{360}{5}e^{-0.05t} + C = -7.2e^{-0.05t} + C$$

$$A = e^{0.10t}\left(-7.2e^{-0.05t} + C\right) = -7.2e^{0.10t}e^{-0.05t} + Ce^{0.10t} = \boxed{-7.2e^{0.05t} + Ce^{0.10t}}$$

general soln

$$0 = A(0) = -7.2 + C \rightarrow C = 7.2$$

$$\boxed{A = -7.2e^{0.05t} + 7.2e^{0.10t} = 7.2\left(e^{0.10t} - e^{0.05t}\right)}$$ IVP solution

$$A(40) = 7.2\left(e^4 - e^2\right) \approx 3,399.054763 \text{ (thousand dollars)}$$

$$\rightarrow \boxed{\$\ 3,399,054.76}$$ to nearest dollar & cent

$$\rightarrow \boxed{\$\ 3,400,000.00}$$ to 3 significant digits

$\rightarrow$ 3.4 million dollars (street language)