

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. [See long instructions on reverse].

$$\textcircled{1} \quad 2 \frac{d^2y}{dt^2} + 16 \frac{dy}{dt} + 50y = 52 \sin 3t, \quad y(0) = 0 = y'(0)$$

- Solve this initial value problem (by hand).
- Your solution should consist of two parts: a decaying oscillation and an oscillation of fixed amplitude. Express the latter (steady state, nondecaying oscillation) in the form  $A \cos(\omega t - \delta)$ , making sure you express  $\delta$  in radians.

$$\textcircled{2} \quad \underline{x}' = \underline{A} \underline{x}, \quad \underline{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad (\text{note } \underline{x}' = \frac{d\underline{x}}{dt}), \quad \underline{A} = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

- Show that 1 and 5 are the eigenvalues of  $\underline{A}$  by using technology to evaluate the appropriate determinant to obtain the characteristic equation for those eigenvalues, and then using technology to solve that equation.
- Using the standard matrix algorithm for solving linear systems of equations (show the system augmented matrix and its rref form for each case, as well as the details of solving the new equivalent system etc), find a basis of eigenvectors for each eigenvalue and join them together to form a basis  $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$  of  $\mathbb{R}^3$ .
- Show by explicit multiplication that  $\underline{B}^{-1} \underline{A} \underline{B}$  is diagonal, where  $\underline{B} = [\underline{b}_1 \underline{b}_2 \underline{b}_3]$ .
- Find the general solution of this DE system using the eigenvector technique. Give your final result in the form  $x_i = \dots$ , etc.
- Solve the system of initial conditions, identifying the system augmented matrix, its rref form and the individual steps in the reduction process. Give your final result in the form  $x_i = \dots$ , etc.
- As a check on your work, write out the three D.E.s of this system ( $x_i' = \dots$ , etc) and back substitute your solution to part e) into these equations and show that they are satisfied.
- As a final check, evaluate  $\underline{x}(0)$  for this solution to make sure it gives the stated initial value.

$$\textcircled{3} \quad \begin{aligned} \frac{dx_1}{dt} &= -4x_1 - 3x_2 & x_1(0) &= 1 \\ \frac{dx_2}{dt} &= 3x_1 - 4x_2 & x_2(0) &= -1 \end{aligned}$$

Solve this initial value problem using the eigenvector technique.

As a check on any errors which might derail your work, keep in mind that  $x_1 = e^{-4t} \sin 3t$  is a possible solution of the DEs (for different initial conditions of course)

This test is to be done without any collaboration. You are encouraged to check all of your work with MAPLE. Although you may access on-line course materials, all your work must be complete calculations in which you supply each step and document your work in a clear organized way. Technology may be used to rref a matrix unless otherwise instructed, to evaluate determinants and to evaluate inverse matrices.

When you have completed the exam, please read and sign the dr bob integrity pledge and place it on top of your answer sheets as a cover page, first side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated."

Signature:

Date: