MAT2705-01/04 03S Test 3 Print Name (Last, First) ______________________________ /100

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. [See long instructions on website].

① $y^{(3)} + 3y'' + 4y' + 12y = f(t)$. Consider first the homogeneous case $f(t) = 0$.

a) Write down the characteristic equation, use technology to factor it and then find its roots.

b) Write down the general solution.

c) Find the solution satisfying the initial conditions $y(0) = 13$, $y'(0) = y''(0) = 0$, by writing down the augmented matrix for the resulting linear system of equations and show the individual steps in its row reduction (you may use technology to do the individual row operations), then check that your solution of this linear system actually solves the system by back substitution. Then give your final answer $y = \cdots$.

d) Your solution $y = y_{exp} + y_{sinusoid}$ should consist of a decaying exponential term and a sinusoidal function term. Did you check that it is correct with MAPLE? Express the sinusoidal function in terms of its amplitude and phaseshift $A\cos(\omega t - \delta)$.

e) How long does it take for the exponential term to decay to 1% of the amplitude A of the sinusoidal function? [$y_{exp}(t) = .01\,A$]
What fraction of the period of the sinusoidal function does this represent (i.e., how many cycles?) ?

f) Make a plot of y, y_{exp}, $y_{sinusoid}$ on the same axes. Print it out and identify the 3 curves by hand marking. Does your graph seem consistent with the results of part e) ? Explain as best you can. [Attach plot to your work.]

g) Now consider the nonhomogeneous case with $f(t) = 30\cos t$. Use the method of undetermined coefficients to find a particular solution y_p. Check by back substitution into the differential equation to see that it is satisfied.

② $mx'' + cx' + kx = F_0 \cos\omega t$ $\quad m = 1,\ c = 10,\ k = 650,\ F_0 = 100$

a) What are the values of the natural decay time τ_0, the natural frequency ω_0, and the quality factor $Q = \omega_0\tau_0$?

b) Find the homogeneous solution of this differential equation (corresponding to $F_0 = 0$).

c) Use the method of undetermined coefficients to find a particular solution of this differential equation (corresponding to the steady state solution) when $\omega = 25$ and express your result in the form $x_{ss}(t) = A\cos(\omega t - \delta)$.

d) Now investigate the possibility of practical resonance of this system by using the method of undetermined coefficients for a general value ω and find the amplitude $A(\omega)$ as a function of ω.

e) Plot this amplitude versus frequency in a window from $\omega = 0$ to many times the natural frequency ω_0. Print it out and attach to your work.

f) Use calculus to find the exact value of ω (and a 3 significant figure approximation) at which the amplitude is maximum. What is the ratio of the maximum amplitude to $A(0)$?

Please reread the test rules at:
http://www.homepage.villanova.edu/robert.jantzen/courses/testquiz/testrules.htm

This test is to be done without any collaboration. If you get stuck on any problem, you may only discuss it with your instructor. You are encouraged to check all of your work with MAPLE.

When you have completed the exam, please read and sign the dr bob integrity pledge and attach it to your answer sheets (staple take home test) as a cover page, first side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated."

Signature:

Date:

3. $\underline{A} = \begin{bmatrix} 11 & -6 & -2 \\ 20 & -11 & -4 \\ 0 & 0 & 1 \end{bmatrix}$ This matrix is diagonalizable and one can find simple integer-valued eigenvectors.

a) Using technology for evaluating determinants, evaluate the characteristic equation for $\underline{A}$, then factor it using technology and find its roots (make sure you enter the matrix correctly).

b) For each eigenvalue find a corresponding basis of its associated eigenspace. Use the standard routine, at most scaling the resulting basis vectors to get integer components. Identify your chosen eigenvectors by $\vec{b}_1 = [x,x,x]$, etc.

c) Combine these together to form a basis of $\mathbb{R}^3$ (order the eigenvalues by increasing value). Write down the basis changing matrix $\underline{B}$ and use technology to find $\underline{B}^{-1}$.

d) Evaluate the matrix product $A_B = B^{-1}AB$ by hand to show that the resulting matrix is diagonal with the expected diagonal values in the expected order.

e) Is your eigenbasis consistent with Maple's choice? How are they related to each other? Explain.

f) Find the new coordinates of the point $[1,2,5]$ with respect to the new basis, and then check that the corresponding linear combination of the basis vectors actually simplifies to $[1,2,5]$.

4. $\underline{A} = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$ a) This matrix has a pair of complex conjugate eigenvalues. Find them and the corresponding complex conjugate pair of eigenvectors $\vec{b}_1$ and $\vec{b}_2$.

b) Is your choice consistent with MAPLE's choice? Explain.