MAT 2705-01/04 03S Test 2 Answers

① a) $\underbrace{\begin{bmatrix} 3 & 1 & -3 & 11 & 10 \\ 5 & 8 & 2 & -2 & 7 \\ 2 & 5 & 0 & -1 & 14 \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\underline{0}}$

b) $\underline{S} = \text{augment}(\underline{A}, \underline{0}) = \begin{bmatrix} 3 & 1 & -3 & 11 & 10 & 0 \\ 5 & 8 & 2 & -2 & 7 & 0 \\ 2 & 5 & 0 & -1 & 14 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ 1 & 0 & 0 & 2 & -3 & 0 \\ 0 & 1 & 0 & -1 & 4 & 0 \\ 0 & 0 & 1 & -2 & -5 & 0 \\ L & L & L & F & F & \end{array}$

c)
$x_1 \quad +2x_4 - 3x_5 = 0 \quad \xrightarrow{d)} \quad x_1 = -2t_1 + 3t_2$

$x_2 \quad -x_4 + 4x_5 = 0 \quad \rightarrow \quad x_2 = t_1 - 4t_2$

$x_3 \quad -2x_4 - 5x_5 = 0 \quad \rightarrow \quad x_3 = 2t_1 + 5t_2$

L: x_1, x_2, x_3 ; F: $x_4 = t_1$, $x_5 = t_2$

$\underline{x} = \begin{bmatrix} -2t_1 + 3t_2 \\ t_1 - 4t_2 \\ 2t_1 + 5t_2 \\ t_1 \\ t_2 \end{bmatrix} \overset{e)}{=} t_1 \begin{bmatrix} -2 \\ 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} 3 \\ -4 \\ 5 \\ 0 \\ 1 \end{bmatrix}$

② a) $c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3 = \vec{0}$

$c_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & -5 & -3 \\ 2 & -3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & -5 & -3 & 0 \\ 2 & -3 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{cccc} c_1 & c_2 & c_3 & \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}$

$c_1 + 2c_3 = 0 \rightarrow c_1 = -2t$

$c_2 + c_3 = 0 \rightarrow c_2 = -t$

$0 = 0$

L L F: $c_3 = t$

$[c_1, c_2, c_3] = [-2t, -t, t] = t[-2, -1, 1]$

$-2\vec{v}_1 - \vec{v}_2 + \vec{v}_3 = 0$ is a linear relationship among these vectors so they are not linearly independent — they are linearly dependent.

[no two are proportional so they span a plane through the origin]

b) $c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3 = \vec{w}$ ... → as before:

$\begin{bmatrix} 1 & -5 & -3 \\ 2 & -3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 0 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 1 & -5 & -3 & -7 \\ 2 & -3 & 1 & 0 \\ 1 & 1 & 3 & 5 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$c_1 + 2c_3 = 3 \rightarrow c_1 = 3 - 2t$

$c_2 + c_3 = 2 \rightarrow c_2 = 2 - t$

$0 = 0 \qquad c_3 = t$

so yes: $\vec{w} = (3-2t)\vec{v}_1 + (2-t)\vec{v}_2 + t\vec{v}_3$ (any value of t will work)

check: $(3-2t)\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + (2-t)\begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix} + t\begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3-2t-10+5t-3t \\ 6-4t-6+3t+t \\ 3-2t+2-t+3t \end{bmatrix} = \begin{bmatrix} -7 \\ 0 \\ 5 \end{bmatrix}$ ✓

③ a) $\underbrace{\begin{bmatrix} 1 & -3 & -3 \\ -1 & 1 & 2 \\ 2 & -3 & -3 \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 6 \\ 4 \\ 9 \end{bmatrix}}_{\underline{b}}$

b) $A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ -1/3 & -1 & -1/3 \\ -1/3 & 1 & 2/3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -3 & 0 & 3 \\ -1 & -3 & -1 \\ -1 & 3 & 2 \end{bmatrix}$

c) $\underline{A}^{-1}[\underline{A}\,\underline{x} = \underline{b}] \rightarrow \underline{A}^{-1}(\underline{A}\,\underline{x}) = \underline{A}^{-1}\underline{b}$

$(\underline{A}^{-1}\underline{A})\,\underline{x} = \underline{I}\,\underline{x} = \underline{x}$

$\underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underline{A}^{-1}\underline{b} = \frac{1}{3}\begin{bmatrix} -3 & 0 & 3 \\ -1 & -3 & -1 \\ -1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \\ 9 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -18+27 \\ -6-12-9 \\ -6+12+18 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 9 \\ -27 \\ 24 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \\ 8 \end{bmatrix}$ technology allowed for evaluating matrix product.

$\boxed{x = 3,\ y = -9,\ z = 8}$

d) changed

$\underbrace{\begin{bmatrix} 2 & -3 & -3 \\ -1 & 1 & 2 \\ 2 & -3 & -3 \end{bmatrix}}_{\underline{A}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 9 \end{bmatrix}$

$\det(\underline{A}) = 0$, A has no inverse so cannot solve in same way.

proceeding with row reduction:

$\begin{bmatrix} 2 & -3 & -3 & 6 \\ -1 & 1 & 2 & 4 \\ 2 & -3 & -3 & 9 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftarrow 0 = 1$

inconsistent system.

(obvious if look at first, third equations before reduction)

so the system has no solution