MAT2705-01/04 03S Test 2 Print Name (Last, First) ________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. [See long instructions on reverse].

①
$$\begin{aligned} 3x_1 + x_2 - 3x_3 + 11x_4 + 10x_5 &= 0 \\ 5x_1 + 8x_2 + 2x_3 - 2x_4 + 7x_5 &= 0 \\ 2x_1 + 5x_2 \qquad - x_4 + 14x_5 &= 0 \end{aligned}$$

a) Write this system of equations in matrix form: $\underline{A}\,\underline{x} = \underline{0}$.

b) Use technology to completely reduce the augmented matrix $\underline{S}$ for this system to its rref form. Write down your result for rref($\underline{S}$).

c) Write out the equivalent system of three equations corresponding to this reduced matrix and identify the leading (L) and free (F) variables.

d) Write down the general solution $x_1 = \ldots, x_2 = \cdots, \ldots$ and also in column matrix form $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$.

e) Express your result as an arbitrary linear combination of column matrices, matrices which form a basis of the solution space.

② $\vec{v}_1 = [1,2,1]$, $\vec{v}_2 = [-5,-3,1]$, $\vec{v}_3 = [-3,1,3]$, $\vec{w} = [-7,0,5]$

a) Is $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ a linearly independent set of vectors? If they are not, find and write down the independent linear relationships among them. Show all of your supporting work and explain how you draw your conclusion.

b) Can $\vec{w}$ be expressed as a linear combination of these vectors? If so, do so, i.e., express $\vec{w}$ as an explicit linear combination of them. Show all supporting work.

③
$$\begin{aligned} x - 3y - 3z &= 6 \\ -x + y + 2z &= 4 \\ 2x - 3y - 3z &= 9 \end{aligned}$$

a) Write this system of equations in matrix form: $\underline{A}\,\underline{x} = \underline{b}$.

b) Use technology to find and write down the inverse matrix A^{-1}.

c) Use the inverse matrix to solve this system (explain what you do). Write down your final result in the form: $x = \cdots, y = \cdots, z = \cdots$.

d) Suppose the first equation is changed to $2x - 3y - 3z = 6$. Use technology to evaluate det(A) for the new system. Report its result. Can we still solve the new system in the same way as before? Explain. Can we solve the new system at all? Explain.