MAT 2500-02/03 02S Takehome Test 3 Print Name (Last, First) ____________________ /100

First read carefully the instructions on the reverse side.

① $\displaystyle\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\, dy\, dx$. Calculate this ~~iterated~~ integral by first reversing the order of integration.

② Evaluate $\displaystyle\iiint_E x^2+y^2\, dV$, where E is the solid region enclosed by the tetrahedron with vertices $(0,0,0)$, $(0,0,1)$, $(2,2,0)$, $(0,2,0)$.

③ Evaluate this integral by converting to spherical coordinates:

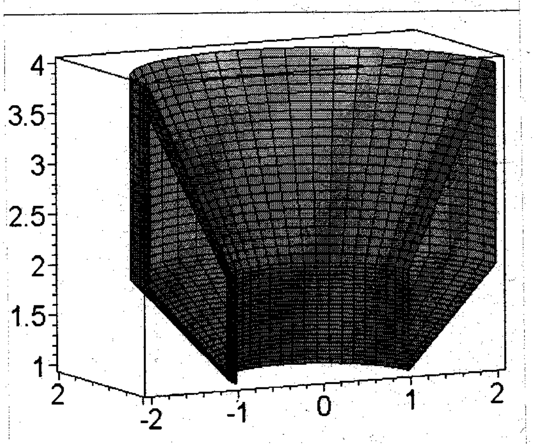
$$\int_0^2 \int_{-\sqrt{4-x^2}}^{0} \int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} z\, dz\, dy\, dx$$

④ $\displaystyle\int_0^1 \int_{\sqrt{y}}^{1} \int_0^{y} xy\, dz\, dx\, dy$ a) Evaluate this integral.

b) Re-express as a triple integral $\iiint \cdots\, dy\, dx\, dz$. [Here you need to analyze the original integral with several diagrams to get a picture of the region of integration and then give several diagrams justifying your re-iteration.]

c) Evaluate the new integral and make sure they agree. If not, check for errors.

⑤



BIZARRO DONUT (half view shown)

Consider the solid region E enclosed by the following surfaces:

$z=\sqrt{x^2+y^2}$ (bottom cone)

$z=2\sqrt{x^2+y^2}$ (top cone)

$x^2+y^2=1$ (inner side cylinder)

$x^2+y^2=4$ (outer side cylinder)

a) Give an r-z halfplane diagram illustrating E as the region of integration in cylindrical coordinates.

b) Find the center of gravity (centroid) of this region using cylindrical coordinates

c) Repeat a) for spherical coordinates.

d) Repeat b) for spherical coordinates. Make sure your results agree.

NOTE: All manipulations of iterated integrals must be accompanied with diagrams and calculations explaining them.